Optimal Exchange Rate Policy*

Oleg Itskhoki
itskhoki@econ.ucla.edu

Dmitry Mukhin
d.mukhin@lse.ac.uk

November 29, 2023

Abstract

We develop a general policy analysis framework for an open economy that features nominal rigidities and financial frictions giving rise to endogenous PPP and UIP deviations. The efficient allocation can be implemented with monetary policy closing the output gap and FX interventions eliminating UIP deviations. When the "natural" real exchange rate is stable, both goals can be achieved solely by monetary policy that fixes the exchange rate — an open-economy divine coincidence. More generally, optimal policy features a managed float/crawling peg complemented with FX forward guidance and macroprudential accumulation of FX reserves, in line with the "fear of floating" observed in the data. Capital controls are not necessary to achieve the frictionless allocation, but they facilitate the extraction of rents in the currency market. Constrained unilateral policies are not optimal from the global perspective, and international cooperation features a complementary use of FX interventions across countries.

*We thank Ozge Akinci, Adrien Auclert, Gianluca Benigno, Emine Boz, Charles Engel, Sebastian Fanelli, Masao Fukui, Luca Fornaro, Guido Lorenzoni and Ivan Werning for insightful discussions, Mark Aguiar, Manuel Amador, Andy Atkeson, Philippe Bacchetta, Suman Basu, Javier Bianchi, Anmol Bhandari, V.V.Chari, Mick Devereux, Mike Golosov, Pierre-Olivier Gourinchas, Lars Hansen, Tarek Hassan, Sebnem Kalemli-Özcan, Rohan Kekre, Robert Kollmann, Karen Lewis, Emi Nakamura, Maury Obstfeld, Fabrizio Perri, Hélène Rey, Ken Rogoff, Dejanir Silva, Jón Steinsson, Ludwig Straub, Steve Wu and seminar/conference participants for useful comments.

1 Introduction

What is the optimal exchange rate policy, and in particular when should the exchange rate be fixed, managed or allowed to freely float? If the optimal exchange rate is managed, is the peg achieved by means of monetary policy, foreign exchange interventions (FXI), or capital controls, or using a mix of these policy instruments? What are the spillovers from unilateral exchange rate policies and are there gains from international cooperation? Is there a trilemma constraint that requires a compromise between inflation and exchange rate stabilization? These classic questions in international macroeconomics are generally difficult to address as the exchange rate is not an immediate policy objective, but rather an endogenous variable with equilibrium linkages in both product and financial markets. At the same time, equilibrium exchange rate behavior features a variety of *puzzles* from the point of view of conventional business cycle models which, in turn, casts doubt on their normative implications (Rogoff 1996, Engel 1996, Obstfeld and Rogoff 2001).

We address these questions by developing a tractable policy analysis framework that is consistent with the major empirical properties of exchange rates, in particular during the episodes of switching between floating and fixed exchange rate regimes (Itskhoki and Mukhin 2021a,b). We focus on a problem of a small open economy with two frictions — nominal rigidities in product markets and imperfect intermediation in segmented financial markets — which correspondingly give rise to frictional deviations from the purchasing power parity (PPP) and the uncovered interest rate parity (UIP). The nominal exchange rate plays a dual role. In the goods market, when prices (or wages) are sticky, it allows for the real exchange rate adjustment and expenditure switching necessary to support the efficient level of output. In the financial market, nominal exchange rate volatility results in a currency risk premium and UIP deviations, as international financial flows must be intermediated by risk-averse market makers exposed to the nominal exchange rate risk. This limits the extent of international risk sharing and results in welfare losses. It is this second policy objective — which emerges from frictional UIP violations and gives rise to effective interventions in currency markets (FXI) — that distinguishes our analysis from the classic trilemma models (Mundell 1963, Fleming 1962).[1]

Following the seminal work of Clarida, Gali, and Gertler (1999, henceforth CGG) and Woodford (2003) for closed economies, we show that the planner's problem attains an intuitive linear-quadratic representation with a welfare loss function that depends on the output gap, inflation and a novel risk-sharing wedge closely related to frictional UIP deviations. Crucially for tractability, we develop a new approximation technique which ensures that the risk premium in the currency market does not drop out from the linearized equilibrium system and affects the first-order dynamics of macroeconomic variables resulting in non-linear state-contingent optimal policies. The planner minimizes welfare losses relative to the first-best allocation subject to the equilibrium conditions in the goods and asset markets and using two policy instruments — the domestic interest rates (monetary policy) and FX interventions in the currency market.

[1] Trilemma models constitute the main policy analysis framework in international macroeconomics following Dornbusch (1976) and Obstfeld and Rogoff (1995). Under trilemma, and in the absence of capital controls, an inward-looking monetary policy uniquely determines the equilibrium exchange rate. In contrast, market segmentation offers the policymaker an additional instrument in the currency market, as it limits the ability of private agents to offset the effect of official FX interventions.

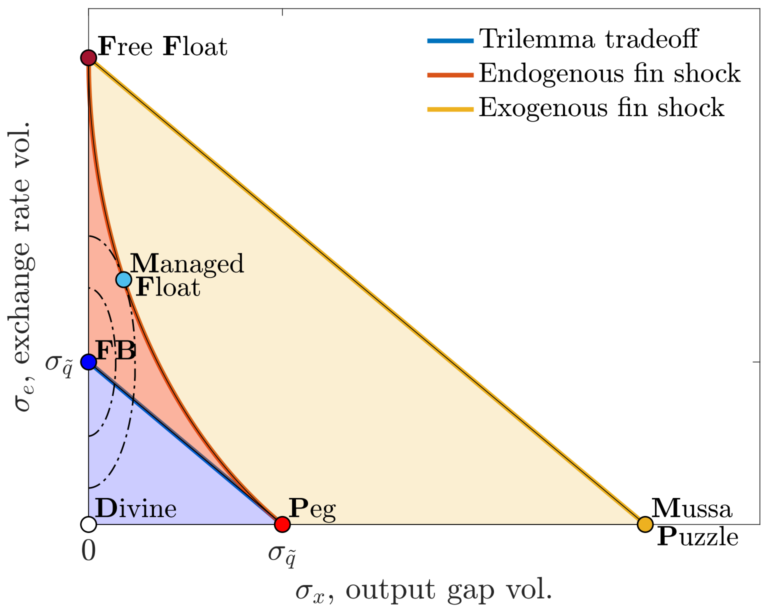


Figure 1: Exchange rate policy tradeoffs

Note: The figure plots the frontiers of output gap and exchange rate volatility, namely menus of (σ_x, σ_e) that can be chosen by monetary policy, in three types of models: (a) classic trilemma models where UIP holds, (b) models with endogenous UIP deviations driven by exchange rate risk, and (c) models with exogenous UIP (or CIP) shocks. FB corresponds to the first best (or a "Friedman float") with $\sigma_x = 0$ and $\sigma_e = \sigma_{\tilde{q}}$, the volatility of the first-best real exchange rate. The line segmented connecting FB and Peg corresponds to the classic Trilemma constraint when UIP holds. Free Float in models with UIP shocks features σ_e that combines macro-fundamental (blue) and financial (red and yellow) exchange rate volatility, and the first best is only feasible when FXI offset financial shocks. Dashed indifference curves are for the welfare loss function, and Managed Float is the optimal monetary policy rule in the absence of FXI. See the text for Divine (coincidence) and Mussa Puzzle points.

We begin by showing how an unconstrained joint use of monetary policy and FX interventions allows the policymaker to implement the first-best allocation. Each policy instrument accommodates a different inefficiency in the economy — with monetary policy targeting inflation and closing the output gap and FXI targeting UIP deviations and eliminating the risk-sharing wedge. The policymaker uses FXI to shift currency risk away from intermediaries' balance sheets thus eliminating the intermediation wedge. However, exchange rate stabilization is not a direct goal of a welfare-maximizing policy. The optimal policy offsets liquidity demand shocks for currency but fully accommodates fundamental macroeconomic shocks to ensure an efficient expenditure switching. The resulting nominal exchange rate tracks the frictionless "natural" real exchange rate.

Figure 1 provides an illustration of the policy tradeoff and the optimal policy choice, comparing our framework with endogenous UIP deviations to two alternative classes of models, namely classic trilemma models without UIP deviations and alternative models with exogenous financial shocks resulting in UIP (or CIP) deviations.[2] Specifically, the figure plots the policy tradeoff in the space of output

[2]The latter class includes models with exogenous UIP shocks (e.g. Devereux and Engel 2002, Kollmann 2005, Farhi and Werning 2012), convenience yield (e.g. Jiang, Krishnamurthy, and Lustig 2021), and financial frictions in the form of balance sheet constraints (e.g. Gabaix and Maggiori 2015, Fanelli and Straub 2021, Basu, Boz, Gopinath, Roch, and Unsal 2020). In Itskhoki and Mukhin (2021a), we show that all such models can be equally successful in explaining the general exchange rate disconnect (i.e., the Free Float point in Figure 1), yet unlike the model with endogenous UIP deviations due to limits to arbitrage these models cannot readily explain the Mussa facts (see Itskhoki and Mukhin 2021b), essential for the optimal exchange

gap and nominal exchange rate volatility. The first best corresponds to a fully eliminated output gap and a nominal exchange rate that under sticky prices must accommodate the volatility of the first-best real exchange rate to ensure efficient expenditure switching. In trilemma models without UIP deviations, a freely floating exchange rate under inward-looking monetary policy and no FXI achieves just that, as suggested by Friedman (1953). More generally, when UIP deviations feature in equilibrium, a laissez-faire float results in excessive exchange rate volatility, which reflects both macro-fundamental and non-fundamental financial volatility, consistent with exchange rate disconnect. In such models, both free floats and full pegs are generally suboptimal and the optimal policy requires an additional use of FXI to offset financial volatility.

Implementing the optimal allocation in the goods and asset markets, in general, requires an unconstrained use of both monetary and FX instruments. However, there exists an important special case when addressing both frictions could be achieved with a nominal exchange rate peg by means of monetary policy alone. We refer to this case as "divine coincidence" in an open economy by analogy with a closed-economy divine coincidence. Indeed, if the natural real exchange rate that ensures the efficient allocation in the goods market is stable, then there is no tradeoff from the point of view of the asset market. In this case, a fixed nominal exchange rate is consistent with efficient expenditure switching in the goods market, and also this eliminates risk in the international financial market allowing for frictionless intermediation. Direct nominal exchange rate targeting is favored over inflation stabilization in this case as it guarantees a unique efficient equilibrium. In Figure 1, Divine (coincidence) corresponds to the situation when $\sigma_{\tilde{q}} = 0$ and the entire blue area collapses to the origin, making the Peg and FB (first best) coincide.[3] While our analysis is consistent with the optimal currency areas logic of Mundell (1961), it identifies not only circumstances when the costs of a currency union are low in the goods market, but also the risk-sharing benefits associated with a fixed exchange rate.

Next, we explore circumstances where financial interventions are constrained, e.g. due to a non-negativity requirement for a central bank foreign reserve holdings or a value-at-risk constraint on the central bank's balance sheet. In this case, there are two independent policy goals — the output gap and the risk-sharing wedge — and monetary policy alone cannot implement the optimal allocation. Instead, optimal policy trades off the output gap and exchange rate stabilization to reduce UIP violations, putting more weight on the latter objective in periods with large capital (out)flows. Managed Float in Figure 1, or a crawling peg, emerges as the second best policy, consistent with the "fear of floating" documented for many developing countries (Calvo and Reinhart 2002). This policy is complemented with both forward guidance and macroprudential accumulation of FX reserves to relax future and past constraints on FX interventions: because of the forward-looking nature of capital flows, future interventions and commitment to future exchange rate stabilization mitigate today's distortions.

We also study the use of capital controls and the ability of the government to use its monopoly power in the currency market to generate rents. When the financial sector is offshore, the policymaker

rate policy analysis. Mussa Puzzle point in Figure 1 illustrates the challenge for models with exogenous financial shocks, where a monetary peg counterfactually absorbs all the floating exchange rate volatility into inflation and the output gap. Propositions 1, 2 and 4 in Section 3 characterize optimal policy corresponding to FB, Divine and Managed Float in the figure.

[3] Note that this is the case both in trilemma models and in a model with endogenous UIP deviations, but not in models with exogenous financial shocks that do not disappear under a fixed nominal exchange rate.

can compete with financial intermediaries for rents (international transfers) that emerge from exogenous shifts in currency demand. If capital controls are available, it is possible to extract these rents without compromising the expenditure switching and risk-sharing goals of the optimal exchange rate policy. Specifically, FXI lean only partially against exogenous shifts in currency demand, leaving non-zero carry trade returns (UIP deviations) on the table, and capital controls are used to eliminate the residual risk-sharing wedge for households. A higher exchange rate volatility lowers the elasticity of currency demand and allows the planner to extract more rents. However, this requires a flexible state-contingent use of capital control taxes and subsidies which may be infeasible. Without capital controls, the policymaker can still use FX interventions to implement the frictionless allocation without incurring financial losses.

Lastly, we extend our model to a global equilibrium with a continuum of small open economies and a dominant currency (dollar) used for international borrowing and lending against other national currencies. We show that the first-best non-cooperative FX policy that closes UIP deviations in all countries implements the globally efficient allocation. In contrast, in the second-best world with constrained FXI, individual countries do not internalize the effect of their policies on the global interest rate r^* and capital outflows from constrained economies. We show that international cooperation calls for strategic complementarity in the use of FXI across countries with a role for central bank FX swap-lines that allow for Pareto-improved allocations. Similarly, the second-best monetary policy with a partial peg to the dollar translates into asymmetric spillovers of U.S. monetary policy and generates a global monetary cycle even when the U.S. accounts for a trivial fraction of global trade.

Our optimal policy results echo many of the themes in Friedman (1953). Discussing the four ways to achieve equilibrium in the currency market, Milton Friedman famously argues in favor of a floating exchange rate and forcefully criticizes capital controls and a nominal peg that distort real allocation in the economy. At the same time, Friedman characterizes FXI as "feasible and not undesirable" as "it may be that private speculation is at times destabilizing for reasons that would not lead government speculation to be destabilizing." This is exactly the mechanism captured by our model. Instead, the differences in our conclusion are quantitative in nature, as Friedman notes that FXI are "largely unnecessary since private speculative transactions will provide currency demand with only minor movements in exchange rates". In other words, the currency supply by intermediaries is sufficiently elastic to accommodate currency demand shocks without large movements in the exchange rate and UIP deviations. In contrast to Friedman who wrote his essay at the height of Bretton Woods, we now have ample evidence following Mussa (1986) that exchange rates are volatile under a free floating regime and feature significant and time-varying UIP deviations, suggesting occasional government interventions may be desirable.

Our modeling framework is related to two lines of work. First, our emphasis on the role of demand for currency in financial markets and the modeling of financial intermediaries follows the tradition of Kouri (1983), Driskill and McCafferty (1987), Dornbusch (1988, Chapter 7) and more recent work by Jeanne and Rose (2002), Blanchard, Giavazzi, and Sa (2005), Camanho, Hau, and Rey (2022), Gourinchas, Ray, and Vayanos (2019), Greenwood, Hanson, Stein, and Sunderam (2020). In contrast to these papers, we embed the financial sector into a realistic general equilibrium model, which is a prerequisite for macroeconomic policy analysis.

Second, we follow Alvarez, Atkeson, and Kehoe (2009) and Gabaix and Maggiori (2015) in assuming that asset markets are segmented. However, guided by the evidence from Itskhoki and Mukhin (2021b), we assume that limits to arbitrage emerge from the risk aversion of intermediaries rather than borrowing constraints or convenience yields, which considerably changes the optimal policy conclusions and distinguishes our analysis from otherwise closely related work by Basu, Boz, Gopinath, Roch, and Unsal (2020, henceforth IPF for Integrated Policy Framework) and other normative papers listed below.[4] The idea that demand for currency is endogenous to the exchange rate regime echoes the classical literature on target zones (Krugman 1991, Svensson 1994) and is in line with the historical evidence from the gold standard (Eichengreen and Flandreau 1997).

Our policy analysis builds on three literatures. First, our analysis incorporates the expenditure switching channel of monetary policy under alternative assumptions on the currency of invoicing as previously studied by Corsetti and Pesenti (2001), Devereux and Engel (2003), Benigno and Benigno (2003), Gali and Monacelli (2005), Goldberg and Tille (2009), Corsetti, Dedola, and Leduc (2010), Engel (2011), Egorov and Mukhin (2023). Second, our analysis is related to recent work on the costs and benefits of FX interventions by Jeanne (2012), Amador, Bianchi, Bocola, and Perri (2019), Cavallino (2019), Fanelli and Straub (2021) and optimal capital controls by Jeanne and Korinek (2010), Bianchi (2011), Costinot, Lorenzoni, and Werning (2014), Farhi and Werning (2016, 2017), Schmitt-Grohé and Uribe (2016). Third, our modeling of the financial channel of monetary policy in an open economy is related to the work of Obstfeld and Rogoff (2002), Kollmann (2004), Benigno (2009), Rey (2013), Fanelli (2017), Kekre and Lenel (2021), Hassan, Mertens, and Zhang (2023), Fornaro (2022), Akinci, Kalemli-Özcan, and Queralto (2022).

2 Modeling Framework

This section introduces the baseline theoretical framework and derives the optimal policy problem. Building on Itskhoki and Mukhin (2021b), we choose the ingredients of the model with an eye to the main empirical properties of exchange rates and intentionally make several strong assumptions to keep the policy problem as simple as possible. We derive a novel linear-quadratic approximation to the planner's problem, which allows us to characterize optimal policies in Section 3. Sections 4 and 5 generalize the setup in several dimensions and consider a number of extensions.

2.1 Setup

We consider a small open economy with tradable and non-tradable goods. There are two frictions — sticky prices and a segmented financial market — that distort the equilibrium allocation, justify government interventions, and give rise to a policy tradeoff. The policymaker can choose the path of nominal interest rates and carry out FX interventions in the currency market.

[4]Our framework agrees with IPF on the first-best allocation and its implementation with monetary policy and FXI. However, the optimal policy away from the first best is different, and in particular the results on the optimality of a monetary peg under divine coincidence and on the second-best managed float/crawling peg by means of monetary policy (at the cost of the output gap) are specific to our model of UIP deviations that are endogenous to the equilibrium exchange rate volatility.

Real sector The households have log-linear preferences over consumption of tradables C_{Tt}, non-tradables C_{Nt} and hours worked L_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\gamma \log C_{Tt} + (1-\gamma)(\log C_{Nt} - L_t) \Big], \tag{1}$$

where γ is the expenditure share on the tradable good capturing the openness of the economy. Households receive labor income $W_t L_t$, firm profits Π_t, and transfers T_t, and can borrow or lend using a one-period risk-free home-currency bond B_t:

$$P_{Tt} C_{Tt} + P_{Nt} C_{Nt} + \frac{B_t}{R_t} = B_{t-1} + W_t L_t + \Pi_t + T_t,$$

where R_t is the gross nominal interest rate.

The endowment of tradable goods Y_{Tt} is exogenous and stochastic generating demand for international risk sharing. The prices of tradables are flexible and satisfy the law of one price:[5]

$$P_{Tt} = \mathcal{E}_t P^*_{Tt},$$

where P^*_{Tt} is the international price of the tradable good and $\mathcal{E}_t$ is the nominal exchange rate in units of home currency for one unit of foreign currency (i.e., an increase in $\mathcal{E}_t$ corresponds to a home depreciation). We assume a stable price level in the foreign country, $P^*_{Tt} = 1$, and therefore the home-currency tradable price tracks the nominal exchange rate, $P_{Tt} = \mathcal{E}_t$.

Output of non-tradables is endogenous and depends on the labor input and productivity shock:

$$Y_{Nt} = A_t L_t.$$

Non-tradable prices are permanently sticky at an exogenous level, $P_{Nt} = 1$, and output is demand determined, $C_{Nt} = Y_{Nt}$. We relax the assumptions on tradable and non-tradable prices in Section 5.[6] Total profits in the economy are given by $\Pi_t = P_{Tt} Y_{Tt} + P_{Nt} Y_{Nt} - W_t L_t$.

The equilibrium in the goods sector is characterized by two optimality conditions. Given that households split their consumption between tradables and non-tradables according to $\gamma P_{Nt} C_{Nt} = (1-\gamma) P_{Tt} C_{Tt}$, and goods prices are $P_{Tt} = \mathcal{E}_t P^*_{Tt} = \mathcal{E}_t$ and $P_{Nt} = 1$, the equilibrium expenditure switching condition is given by:

$$\frac{\gamma}{1-\gamma} \frac{C_{Nt}}{C_{Tt}} = \frac{\mathcal{E}_t P^*_{Tt}}{P_{Nt}} = \mathcal{E}_t. \tag{2}$$

The relative demand for goods depends on their relative price, $\mathcal{E}_t P^*_{Tt}/P_{Nt}$, which under fully sticky prices of non-tradables is equal to the nominal exchange rate $\mathcal{E}_t$. The optimal consumption-savings

[5] Exogenous terms of trade due to a homogenous tradable good eliminate the beggar-thy-neighbour policy motive that typically complicates the normative analysis (see Corsetti and Pesenti 2001) and make international risk sharing depend on the structure of asset markets despite logarithmic preferences (*cf.* Cole and Obstfeld 1991).

[6] We focus on the fully sticky price case as a limiting benchmark which simplifies the analysis by avoiding an additional dynamic equation, yet maintains all the qualitative tradeoffs of a more general environment discussed in Section 5.1. By having price stickiness (or, equivalently, wage stickiness) only in the non-tradable sector, we avoid the need to choose between PCP/LCP/DCP frameworks, which we analyze in Section 5.2 along with imperfect substitutability between home and foreign tradables and incomplete exchange rate pass-through into tradable prices.

decision of households is described by a standard Euler equation:

$$\beta R_t \mathbb{E}_t \frac{C_{Nt}}{C_{Nt+1}} = 1, \tag{3}$$

and depends on the nominal interest rate R_t set by the policymaker. Finally, the optimality condition for labor supply, $C_{Nt} = W_t/P_{Nt} = W_t$, determines the equilibrium nominal wage.

Financial sector While the equilibrium in the goods market is conventional to open-economy sticky-price models, our analysis deviates from this literature by introducing segmentation in global asset markets. In particular, we assume that home households have access exclusively to local-currency bonds, and hence all international capital flows have to be intermediated by specialized financial traders.[7]

Household demand for the home-currency bond B_t reflects fundamental macroeconomic forces and shapes the equilibrium path of net exports and net foreign assets. Additionally, there are three types of agents that can trade home and foreign currency bonds in the international financial market — the government, noise traders and intermediaries (arbitrageurs) — all residing in the home economy (see Appendix Figure A1). For these agents who have access to foreign-currency (dollar) saving and borrowing, the dollar bond is in a perfectly elastic international supply at an exogenous interest rate R_t^*. Section 4 considers extensions that allow for foreign intermediaries and noise traders resulting in cross-border financial income transfers, as well as an endogenize R_t^* in a multi-country global economy.

Each period, arbitrageurs choose a zero capital portfolio (D_t, D_t^*) such that $D_t/R_t = -\mathcal{E}_t D_t^*/R_t^*$, where $1/R_t$ and $1/R_t^*$ are prices of the two bonds. The dollar net income of arbitrageurs from such a carry trade is given by $\pi_{t+1}^{D*} = D_t^* - D_t/\mathcal{E}_t = \tilde{R}_{t+1}^* \cdot \frac{D_t^*}{R_t^*}$, where $\tilde{R}_{t+1}^* = R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ is a one-period return on one dollar holding of a carry trade portfolio. This income is transferred lump-sum to households.

Arbitrageurs choose their portfolio to maximize min-variance preferences, $\mathbb{E}_t\big[\Theta_{t+1}\pi_{t+1}^{D*}\big] - \frac{\omega}{2}\mathrm{var}_t\big(\pi_{t+1}^{D*}\big)$, where $\Theta_{t+1} = \beta \frac{C_{Tt}}{C_{Tt+1}} \frac{P_{Tt}^*}{P_{Tt+1}^*}$ is the stochastic discount factor (SDF) of home households and ω is the risk aversion parameter of arbitrageurs. The second term in the objective function is the additional risk penalty capturing the intermediation friction that creates limits to arbitrage. The optimal portfolio choice satisfies:

$$\frac{D_t^*}{R_t^*} = \frac{\mathbb{E}_t\big[\Theta_{t+1}\tilde{R}_{t+1}^*\big]}{\omega\sigma_t^2}, \tag{4}$$

where $\sigma_t^2 \equiv \mathrm{var}_t(\tilde{R}_{t+1}^*) = R_t^2 \cdot \mathrm{var}_t(\mathcal{E}_t/\mathcal{E}_{t+1})$ is a measure of the nominal exchange rate volatility. As $\omega \to 0$, the risk-absorption capacity of arbitrageurs increases unboundedly, and the uncovered interest rate parity (UIP) holds in equilibrium, $\mathbb{E}_t[\Theta_{t+1}\tilde{R}_{t+1}^*] = 0$.[8]

[7]Similarly to the assumption of fully rigid prices, complete segmentation of asset markets substantially simplifies our analysis by limiting the number of dynamic equations. Yet, our main insights remain valid under a more realistic form of segmentation where households can trade foreign currency bonds subject to additional transaction costs and gradual portfolio adjustment (see Aiyagari and Gertler 1999, Bacchetta, Tieche, and Van Wincoop 2020, Fukui, Nakamura, and Steinsson 2023).

[8]More precisely, in this limit, the household SDF Θ_{t+1} prices the exchange rate risk, and the expected return on the carry trade is given by $\mathbb{E}_t\tilde{R}_{t+1}^* = R_t^* \cdot \mathrm{cov}_t(\Theta_{t+1}, \mathcal{E}_t/\mathcal{E}_{t+1})$, a property of the optimal risk sharing. Note that $\omega > 0$, which gives rise to frictional risk premium, shall not be interpreted as greater risk aversion of arbitrageurs relative to households, as households cannot/do not hold exchange rate risk in equilibrium, while arbitrageurs hold it in a concentrated way.

Noise traders also hold a zero capital portfolio (N_t, N_t^*) of home and foreign-currency bonds, such that $N_t/R_t = -\mathcal{E}_t N_t^*/R_t^*$, and N_t^* is an exogenous liquidity demand shock for foreign currency that is uncorrelated with macroeconomic fundamentals. A positive N_t^* means that noise traders short home-currency bonds to buy foreign-currency bonds, and vice versa. Noise traders' net income and losses are transferred to the households. Although difficult to measure in the data, these shocks are necessary to match the disconnect properties of the exchange rate. Importantly, our normative results do not require that N_t^* is pure noise, and go through when one assumes that currency demand is driven by household preference shocks for foreign-currency bonds (Itskhoki and Mukhin 2022).

Finally, the government holds a portfolio (F_t, F_t^*) of home- and foreign-currency bonds with the net value of the portfolio given by $F_t/R_t + \mathcal{E}_t F_t^*/R_t^*$. Changes in F_t and F_t^* correspond to open market operations of the government. The net government income and losses are also transferred to the households. Therefore, net transfers of income to the households from financial transactions of the government, noise traders and arbitrageurs are equal to:

$$T_t = \left(F_{t-1} - \frac{F_t}{R_t}\right) + \mathcal{E}_t \left(F_{t-1}^* - \frac{F_t^*}{R_t^*}\right) + \mathcal{E}_t \tilde{R}_t^* \cdot \frac{N_{t-1}^* + D_{t-1}^*}{R_{t-1}^*}.$$

Financial market clearing requires that the home-currency bond positions of all four types of agents balance out, $B_t + N_t + D_t + F_t = 0$. We define B_t^* to be the net foreign asset (NFA) position of the home country, expressed in foreign currency, such that:

$$\frac{B_t^*}{R_t^*} = \frac{1}{\mathcal{E}_t}\frac{B_t + F_t}{R_t} + \frac{F_t^*}{R_t^*}.$$

Thus, home NFA is the value of the combined position of home households and the government, as the remaining agents in the financial market hold zero value portfolios, albeit exposed to currency risk.

Using this definition and the zero value portfolios of noise traders and arbitrageurs, we rewrite the financial market clearing condition as:

$$B_t^* = F_t^* + N_t^* + D_t^*. \tag{5}$$

In other words, the NFA position of the country equals the combined foreign-currency bond position in the financial market. That is, currency market equilibrium requires that currency supply B_t^* from accumulated NFA equals aggregate currency demand, $F_t^* + N_t^* + D_t^*$.[9]

Equilibrium Two international conditions — the country budget constraint and international risk sharing — complete the description of the equilibrium system. To derive the country budget constraint, we substitute the expressions for profits Π_t and financial transfers T_t into the household budget constraint. This yields:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt}, \tag{6}$$

[9]Since B_t^*, N_t^*, D_t^* and possibly F_t^* can take positive and negative values, who supplies and demands currency in the market can change. Positive (negative) values of N_t^*, D_t^* and F_t^* correspond to currency demand (supply), and vice versa for B_t^*.

where the right-hand side is net exports expressed in dollars (or in terms of tradables, since $P^*_{Tt} = 1$). Intuitively, trade surpluses lead to the accumulation of net foreign assets — a macro-fundamental source of currency supply to the home market.

To derive the international risk-sharing condition, we combine household optimality (2) and (3) with the equilibrium conditions in the financial market (4) and (5). This results in:[10]

$$\beta R^*_t \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega\sigma_t^2 \frac{B^*_t - N^*_t - F^*_t}{R^*_t}, \qquad \text{where} \quad \sigma_t^2 = R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big). \tag{7}$$

If households had direct access to dollar bonds, then a conventional Euler equation $\beta R^*_t \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1$ would hold. Instead, household positions need to be intermediated by the financial sector which charges a risk premium — a risk-sharing wedge. This risk premium depends both on the size of the currency exposure of arbitrageurs, $D^*_t = B^*_t - F^*_t - N^*_t$, and the price of risk $\omega\sigma_t^2$ per dollar of the exposure.

Currency outflows — due to both fundamental ($B^*_t < 0$) and non-fundamental ($N^*_t > 0$) reasons — require intermediation ($D^*_t < 0$) and expose arbitrageurs to currency depreciation risk, resulting in an equilibrium risk premium and a risk-sharing wedge.[11] Greater exchange rate volatility σ_t^2 increases the price of risk and the resulting risk-sharing wedge for given gross currency positions. A policymaker can intervene either by reducing expected exchange rate volatility or by absorbing the currency risk into the government balance sheet with FX interventions ($F^*_t \downarrow$), as we study in the next section.

Finally, we define the equilibrium in this economy. Given the stochastic path of exogenous shocks $\{A_t, Y_{Tt}, R^*_t, N^*_t\}$, sticky non-tradable prices $P_{Nt} \equiv 1$, and the path of policies $\{R_t, F_t, F^*_t\}$, an equilibrium vector $\{C_{Nt}, C_{Tt}, B^*_t, D^*_t, \mathcal{E}_t\}$ and the implied $\{\sigma_t^2\}$ solve the dynamic system (2)–(7) with the initial condition B^*_{-1} and the transversality condition $\lim_{T\to\infty} B^*_T / \prod_{t=0}^T R^*_t = 0$.[12] Note that Ricardian equivalence does not hold vis-à-vis foreign currency position F^*_t, as households cannot directly hold foreign currency bonds. As a result, both the country's NFA position B^*_t and the arbitrageurs' currency exposure $D^*_t = B^*_t - N^*_t - F^*_t$ are endogenous state variables of the equilibrium system. In contrast, the model features Ricardian equivalence for home-currency bonds — a change in F_t merely crowds out private B_t, and hence it is not a state variable for the equilibrium allocation.

[10] Household Euler equation (3) together with the expenditure allocation (2) imply $\mathbb{E}_t\{\Theta_{t+1} R_t \mathcal{E}_t/\mathcal{E}_{t+1}\} = 1$. Arbitrageurs' portfolio choice (4) implies $\mathbb{E}_t\{\Theta_{t+1}[R^*_t - R_t\mathcal{E}_t/\mathcal{E}_{t+1}]\} = \omega\sigma_t^2 D^*_t / R^*_t$, which is the frictional UIP deviation. Adding the two expressions together to eliminate R_t and using the financial market clearing (5) to substitute for D^*_t results in (7). Note from this derivation that the risk-sharing wedge is equal to the frictional UIP deviation, and they both disappear as $\omega\sigma_t^2 \to 0$.

[11] Frictional risk premium takes the form of a UIP shock that is accommodated by a jump depreciation followed by an expected appreciation, or vice versa. This distorts the path of tradable prices and tradable consumption, resulting in suboptimal risk sharing. While our analysis focuses on the distortion to consumption, similar considerations apply in a model with investment and the associated frictional risk premium that distorts the required rate of return on investment away from R^*_t.

[12] Note that (4) is redundant given (7) and (5). Hence, we have four independent equilibrium conditions, (2)–(3) and (6)–(7), to solve for four endogenous variables $\{C_{Nt}, C_{Tt}, B^*_t, \mathcal{E}_t\}$, and a side equation (5) to solve for D^*_t. The other endogenous variables $\{W_t, L_t, Y_{Nt}, B_t\}$ are recovered from static equilibrium conditions outlined above. Specifically, from the goods market clearing and labor supply $Y_{Nt} = C_{Nt}$ and $W_t = C_{Nt}$; from production function $L_t = Y_{Nt}/A_t$; and B_t can be backed out from (B^*_t, F^*_t, F_t) given the definition of NFA B^*_t.

2.2 Policy problem

In our baseline analysis, we focus on the Ramsey problem of choosing a sequence of government policies that maximize welfare under commitment. Given the equilibrium definition above, the government chooses a feasible path of monetary policy and FX interventions, $\{R_t, F_t^*\}$, that maximizes household welfare (1).[13] We set up the exact non-linear policy problem in Appendix A1, which allows for a characterization of the first-best allocation and the policies that decentralize it. To make progress for the main cases of interest, where the first-best allocation is not feasible given the available policy instruments, we work with a linear-quadratic approximation to the policy problem around the first-best allocation.

In this section, we derive the approximate policy problem. In doing so, we address two major challenges associated with the transition to a linear-quadratic environment. The first challenge relates to the quadratic approximation of the welfare function in an open economy, and in particular where the best possible risk sharing is not full insurance, as the international financial market is incomplete and features risk free bonds only. The second challenge arises due to the risk-sharing friction driven by a time-varying risk premium in the currency market that disappears in conventional linear approximations. Our approach ensures that the risk-sharing friction remains in the linear-quadratic environment, preserving the key policy tradeoff between output gap stabilization and international risk sharing.

First-best allocation The first-best allocation is the path of tradable and non-tradable consumption and labor, which we denote with tildes $\{\tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_t\}$, that maximizes the household welfare in (1) subject to the country budget constraint (6) and the non-tradable production possibility frontier $C_{Nt} = Y_t = A_t L_t$, taking as given the path of shocks $\{Y_{Tt}, A_t, R_t^*\}$ and the initial net foreign assets B_{-1}^*, as well as the No-Ponzi-Game Condition (NPGC) for B_∞^*. This problem abstracts from both the sticky price friction in the goods market and the intermediation friction in the financial market. Furthermore, the local planner takes as given the structure of the international financial market which provides a perfectly elastic supply of dollar risk-free bonds at an exogenous interest rate R_t^*.

Given the log-linear utility (1), the first-best allocation features a constant labor supply $\tilde{L}_t = 1$ yielding $\tilde{C}_{Nt} = A_t$, and a path of $\tilde{C}_{Tt}$ that solves a frictionless Euler equation $\beta R_t^* \mathbb{E}_t\{C_{Tt}/C_{Tt+1}\} = 1$ together with the country budget constraint (6). Therefore, $\tilde{C}_{Tt}$ is a function of shocks $\{Y_{Tt}, R_t^*\}$ and the initial net foreign assets B_{-1}^*. The first-best path of NFA satisfies the country budget constraint (6), that is $\tilde{B}_t^* = R_t^*(\tilde{B}_{t-1}^* + Y_{Tt} - \tilde{C}_{Tt})$.

With fully sticky non-tradable prices, the decentralization of the first-best allocation involves a path of nominal wages $\tilde{W}_t = A_t$ to ensure the first-best labor supply, and a path of the nominal exchange rate

$$\tilde{\mathcal{E}}_t = \tilde{\mathcal{Q}}_t = \frac{\gamma}{1-\gamma} \frac{\tilde{C}_{Nt}}{\tilde{C}_{Tt}} \tag{8}$$

[13] The government does not take the welfare of arbitrageurs and noise traders into account, as these agents pass on all their financial incomes and losses to the households. Yet, their behavior — namely, exogenous currency demand of noise traders and endogenous inelastic currency supply by arbitrageurs — affects the equilibrium allocation in the financial market via a wedge in the risk-sharing condition (7). This is akin to the behavior of a monopolist in the goods market that creates a markup wedge and passes on all profits back to the households.

to ensure the first-best relative price and expenditure allocation between tradables and non-tradables in (2). Equation (8) defines $\tilde{\mathcal{Q}}_t$ which we refer to as the first-best, or *natural*, real exchange rate. It is the value of international relative prices that ensures the optimal expenditure allocation between tradables and non-tradables in our economy.[14]

Second-order approximation to the welfare function We evaluate welfare losses due to a departure of the equilibrium allocation from the first best. To do so, we derive a second-order approximation to the objective function in (1) around a non-stochastic steady state and evaluate the welfare loss relative to the first-best allocation $\{\tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_t\}$ characterized above.

To this end, we introduce two wedges central to our analysis — the output gap x_t and the risk-sharing wedge z_t — defined by:

$$x_t \equiv \log C_{Nt} - \log \tilde{C}_{Nt} \qquad \text{and} \qquad z_t \equiv \log C_{Tt} - \log \tilde{C}_{Tt}. \tag{9}$$

The output gap x_t emerges as a result of sticky non-tradable prices, and it measures the gap in non-tradable consumption relative to $\tilde{C}_{Nt} = A_t$. This also corresponds to the departure of labor supply $L_t = C_{Nt}/A_t$ from $\tilde{L}_t = 1$. The risk-sharing wedge z_t is the result of a violation of the first-best risk sharing. Specifically, an intermediation wedge in (7) causes a risk-sharing wedge. Note that all feasible paths of C_{Tt}, and hence z_t, must still satisfy the country budget constraint (6).

To make the analysis tractable, the innovation of our approach is to focus only on budget-feasible allocations $\{C_{Tt}, C_{Nt}, L_t\}$ that satisfy the production possibilities frontier for non-tradables, $C_{Nt} = A_t L_t$, and the country budget constraint for tradables, that is (6) together with the NPGC for B^*_∞ and given B^*_{-1}. For every such allocation that results in wedges x_t and z_t defined in (9), the welfare loss relative to the first best is given by:[15]

$$\text{Loss} = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[\gamma z_t^2 + (1-\gamma) x_t^2\big], \tag{10}$$

where the weight on the risk-sharing wedge equals γ, a measure of the degree of openness of the economy. We provide a formal derivation of (10) in Appendix A2 where we introduce a novel Lagrangian-based method to derive a second-order welfare loss function for any feasible allocation relative to the first best, which by construction features no first-order terms.

The path of the risk-sharing wedge z_t must be consistent with the budget constraint (6), which in deviations from the first best is given by:

$$\beta b_t^* - b_{t-1}^* = -z_t, \tag{11}$$

where $b_t^* \equiv (B_t^* - \tilde{B}_t^*)/\bar{Y}_T$ is the deviation of NFA from its first-best path scaled by the steady-state level

[14] Formally, the real exchange rate is $\mathcal{E}_t P_t^*/P_t = \mathcal{E}_t^{1-\gamma}$ (with $P_t^* = P_{Tt}^* = 1$ and $P_t = P_{Tt}^{\gamma} P_{Nt}^{1-\gamma} = \mathcal{E}_t^{\gamma}$), while $\mathcal{E}_t P_{Tt}^*/P_{Nt} = \mathcal{E}_t$ is the relative price of non-tradables; the two are proportional to each other in logs. More generally, in every economy, one can define a relevant concept for the first-best real exchange rate that, given goods market clearing condition, ensures an efficient expenditure allocation between home and foreign goods.

[15] All our approximations are around a steady state with $\bar{B}^* = \bar{F}^* = \bar{N}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$; with tradable endowment $\bar{Y}_T$, non-tradable productivity $\bar{A}$, and exchange rate $\bar{\mathcal{E}} = \frac{\gamma}{1-\gamma}\frac{\bar{A}}{\bar{Y}_T}$, resulting in $\bar{L} = 1$, $\bar{C}_N = \bar{A}$, $\bar{C}_T = \bar{Y}_T$, $\overline{NX} = 0$.

of tradable output $\bar{Y}_T$, and $\beta\bar{R}^* = 1$ in steady state. The initial condition is $b^*_{-1} = 0$ (as $\tilde{B}^*_{-1} = B^*_{-1}$), the NPGC is $\lim_{t\to\infty}\beta^t b^*_t = 0$, and thus $z_t = b^*_t = 0$ for all $t \geq 0$ is a feasible allocation corresponding to the first-best risk sharing. Note that z_t acts simultaneously as the risk-sharing wedge and as the deviation of net exports from their first-best path, $z_t = -(nx_t - \widetilde{nx}_t)$ where $\widetilde{nx}_t \equiv (Y_{Tt} - \tilde{C}_{Tt})/\bar{Y}_T$. Cumulated deviations of net exports z_t result in deviations of NFA b^*_t, as summarized by (11).

First-order approximation to the equilibrium system Minimizing the welfare loss (10) subject to the budget constraint (11) alone poses no policy tradeoff as $x_t = z_t = 0$ for all $t \geq 0$ is a budget-feasible allocation. In addition to the budget constraint (11), the first-order approximation to the exact equilibrium system (2)–(7) involves two additional conditions — one that characterizes equilibrium in the goods market and the other that characterizes equilibrium in the financial market.

In the goods market, the expenditure allocation condition (2) can be written in log deviations as:

$$e_t = \tilde{q}_t + x_t - z_t, \tag{12}$$

where $e_t = \log\mathcal{E}_t$ and $\tilde{q}_t = \log\tilde{\mathcal{Q}}_t$ is the first-best real exchange rate defined in (8), and the two wedges x_t and z_t as defined in (9). Given sticky prices, the nominal exchange rate must accommodate movements in the first-best real exchange $\tilde{q}_t$, otherwise one or both wedges open up. Indeed, if the relative price of non-tradables is off its first-best level, either tradable or non-tradable consumption (or both) must deviate from their first-best levels as well. Equation (12) captures the locus of possible equilibrium allocations in the goods market shaped by expenditure switching between tradables and non-tradables.[16]

The remaining condition characterizes equilibrium in the financial (currency) market. The risk-sharing friction emphasized in (7) corresponds to the risk premium charged by arbitrageurs for intermediating currency trades and holding the associated exchange rate risk. In conventional linear approximations, risk premia go to zero with second moments such as σ_t^2. We consider an alternative point of approximation in which risk premia remain first-order objects and, hence, affect first-order dynamics of the equilibrium system. To this end, we let the risk aversion parameter ω to increase as σ_t^2 decreases, keeping the price of risk $\omega\sigma_t^2$ non-zero in the limit. We provide formal details in Appendix A2, where we show that our first-order approximation to (7) results in:

$$\mathbb{E}_t\Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2(n^*_t + f^*_t - b^*_t) \qquad \text{with} \quad \bar{\sigma}_t^2 = \text{var}_t(e_{t+1}), \tag{13}$$

where $\bar{\omega} \equiv \omega\bar{Y}_T/\beta$, $f^*_t \equiv F^*_t/\bar{Y}_T$ are FXI scaled by tradable output, and $n^*_t \equiv (N^*_t - \tilde{B}^*_t)/\bar{Y}_T$ is a combined exogenous currency demand shock.[17] Like a conventional first-order approximation, our approximation scales linearly with the size of exogenous shocks that drive n^*_t in (13) and $\tilde{q}_t$ in (12). However, due to an unconventional point of approximation in which the risk-bearing capacity of the

[16] Rewriting (12) as $x_t = z_t + (e_t - \tilde{q}_t)$ one can interpret this condition as follows: a capital outflow shock $z_t < 0$ must be accommodated by either a depreciation $e_t > \tilde{q}_t$ or an output gap $x_t < 0$ if the exchange rate fails to adjust.

[17] Note that n^*_t features both noise trader demand for foreign currency ($N^*_t > 0$) net of supply of foreign currency accumulated from the first-best path of net exports (that is, NFA $\tilde{B}^*_t > 0$); of course, these variables can take both positive and negative values with corresponding interpretations.

financial sector $1/\omega \to 0$, the equilibrium system is *non*-linear in shocks (and hence state variables), and in particular features time-varying volatility that affects first-order equilibrium dynamics.

Condition (13), together with the budget constraint (11), characterizes the equilibrium path of tradable consumption relative to its first-best level, $z_t \equiv \log(C_{Tt}/\tilde{C}_{Tt})$, from the point of view of households. Alternatively, it also determines the path of UIP deviations from the point of view of the financial market (recall conditions (4) and (5)). Indeed, we have that the UIP deviation equals:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta z_{t+1}, \tag{14}$$

where $i_t - i_t^* = \log(R_t/R_t^*)$.[18] Therefore, the risk-sharing wedge in (13), $\bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*)$, is also the frictional UIP wedge. It is a first-order object that comoves with the intermediated demand for currency, $n_t^* + f_t^* - b_t^*$, and with the unit price of the exchange rate risk, $\bar{\omega}\bar{\sigma}_t^2$. Thus, variation in the conditional exchange rate volatility $\bar{\sigma}_t^2$ is both an equilibrium outcome and has a direct first-order feedback into equilibrium dynamics.

In Appendix A2, we prove two additional results. First, the dynamic system (11)–(13) provides an accurate first-order approximation to the exact equilibrium dynamics. That is, taking the path of exogenous shocks $\{\tilde{q}_t, n_t^*\}$ and policies $\{x_t, f_t^*\}$, this system characterizes the path of endogenous equilibrium outcomes $\{z_t, b_t^*, e_t, \bar{\sigma}_t^2\}$. Note that we take the output gap x_t as the policy variable since it is directly controlled by the monetary policy instrument i_t. Second, we prove that minimizing the second-order welfare loss in (10) with respect to $\{x_t, f_t^*, z_t, b_t^*, e_t\}$ and subject to the linearized equilibrium system (11)–(13) results in a first-order accurate description of the optimal policies in the exact non-linear problem. While the equilibrium system is non-linear in the path of shocks and state variables due to the presence of $\bar{\sigma}_t^2$ in (13), the policy problem scales proportionally with the general magnitude of shocks, and in this narrow sense one may refer to this policy problem as linear-quadratic.[19]

Our approach to approximation combines analytical tractability of a first-order approximation with the ability to match the equilibrium size and dynamics of risk premia, which allows us to study the co-dynamics of macroeconomic and financial variables and their interaction in shaping the optimal policies. Alternative approaches include a full non-linear solution and a higher-order approximation. We expect these approaches to yield comparable conclusions, as long as they also match the size and dynamic properties of the risk premium, which is quantitatively large and consequential for macroeconomic allocations. At the same time, exact methods are non-analytical and computationally costly in our optimal policy environment that, as we show below, requires commitment and dynamic follow-through on policy promises, which increases the size of the state space.

[18] From the frictionless international Euler equation, $i_t^* = \log(R_t^*/\bar{R}^*) = \mathbb{E}_t \Delta\tilde{c}_{Tt+1}$. The home Euler equation (3) together with (2), in turn, implies $i_t = \log(R_t/\bar{R}) = \mathbb{E}_t\{\Delta c_{Tt+1} + \Delta e_{t+1}\}$. Subtracting one from the other, and using the fact that $z_t \equiv c_{Tt} - \tilde{c}_{Tt}$, yields the UIP expression in the text. Note also that the equilibrium path of the local interest rate can be recovered from (3) as $i_t = \tilde{r}_t + \mathbb{E}_t \Delta x_{t+1}$, where $x_t = c_{Nt} - a_t$ and $\tilde{r}_t = \mathbb{E}_t \Delta a_{t+1}$ is the natural real interest rate.

[19] Formally, we let ν scale all shocks $\{A_t, Y_{Tt}, R_t^*, N_t^*\}$ in the exact non-linear economy with $1/\omega$ scaled by ν^2 to keep the unit price of risk $\omega\sigma_t^2$ stable. Then, the linearized system (11)–(13) characterizes the first-order component of the non-linear system, which scales proportionally with ν, while the welfare loss in (10) scales proportionally with ν^2. The equilibrium price of risk $\omega\sigma_t^2$ and the optimal policy lean (δ_t in Theorem 1 below) do not scale with ν (are zero order in ν), but are generally time-varying. For related but different approaches to approximation see Judd and Guu (2001), Hansen and Sargent (2011), Hansen and Miao (2018), Bhandari, Evans, Golosov, and Sargent (2017), Caballero and Farhi (2018), Caramp and Silva (2023).

3 Optimal Policies

Given the results in Section 2.2, we restate here the baseline Ramsey policy problem (10)–(13) of a small-open economy policymaker in deviations from the first-best allocation:

$$\min_{\{x_t, f_t^*, z_t, e_t, b_t^*, \bar{\sigma}_t^2\}} \frac{1}{2}\,\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \beta^t\big[\gamma z_t^2 + (1-\gamma)x_t^2\big] \tag{15}$$

$$\text{subject to} \qquad \beta b_t^* = b_{t-1}^* - z_t,$$

$$e_t = \tilde{q}_t + x_t - z_t$$

$$\mathbb{E}_t \Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*) \quad \text{with } \bar{\sigma}_t^2 = \mathrm{var}_t(e_{t+1}),$$

and potential constraints on FXI $f_t^* \in \mathcal{F}_t$, the initial condition $b_{-1}^* = 0$ and the transversality condition $\lim_{t\to\infty}\beta^t b_t = 0$. The policymaker directly controls the path of the output gap and FXI, $\{x_t, f_t^*\}$. The path of policies may be restricted by additional constraints $f_t^* \in \mathcal{F}_t$, e.g. a non-negativity constraint on FX reserves $f_t^* \geq 0$ or a value-at-risk constraint $\bar{\sigma}_t \cdot |f_t^*| \leq \bar{\alpha}$.

All exogenous shocks affecting equilibrium dynamics are summarized by two variables — the natural (first-best) real exchange rate $\tilde{q}_t$ defined in (12) and the exogenous net currency demand shock n_t^* defined in (13). In particular, $\tilde{q}_t$ is a sufficient statistic for all macroeconomic shocks $\{A_t, Y_{Tt}, R_t^*\}$ that shape the first-best path of tradable and non-tradable consumption. In turn, n_t^* summarizes currency demand shocks of noise traders N_t^* and households $\tilde{B}_t^*$, with the latter shaped by the path of the first-best $\widetilde{NX}_t \equiv Y_{Tt} - \tilde{C}_{Tt}$. Departures from the first-best path of tradable consumption result in risk-sharing wedges $z_t \equiv \log(C_{Tt}/\tilde{C}_{Tt})$, which via (11) lead to deviations of NFA b_t^* that also feed back via (13) as an additional source of endogenous currency supply (or demand, if negative).

The goal of the policy (15) is to minimize deviations from the first-best allocation — namely, eliminate to the extent possible the output gap x_t and the risk-sharing wedge z_t, with the relative weight on the latter given by the openness of the economy γ. Policies shape the equilibrium path of the exchange rate e_t, and thus its conditional volatility $\bar{\sigma}_t^2$, which in turn affects the dynamics of the equilibrium system via (13). We note that a particular level or volatility of the exchange rate is not a policy goal in itself. Nonetheless, the exchange rate e_t emerges as the key equilibrium variable linking the financial and the goods markets, putting it at the center of the policy tradeoff. When prices are sticky, movements in the nominal exchange rate are necessary to accommodate the adjustment of relative prices in the goods market (12). However, volatility of the exchange rate is also a source of the risk-sharing wedge in the financial market with imperfect intermediation (13).[20]

Relaxed Trilemma An important feature of the model is that the planner can sidestep the standard trade-off between an independent monetary policy and a managed exchange rate. Even in the absence of capital controls, the government can choose the path of the output gap x_t with an inward-looking in-

[20]Formally, if monetary policy stabilizes the output gap, $x_t = 0$, then from (12) the nominal exchange rate must equal $e_t = \tilde{q}_t - z_t$. This, in general, results in $\bar{\sigma}_t^2 = \mathrm{var}_t(e_{t+1}) > 0$, and hence a non-zero risk-sharing wedge $z_t \neq 0$ from (13). Conversely, optimal risk sharing $z_t = 0$ can only be achieved with $\sigma_t^2 = 0$ in the absence of FX interventions ($f_t^* = 0$), which in turn requires $e_t = \tilde{q}_t + x_t = 0$, and thus in general a non-zero output gap, $x_t = -\tilde{q}_t$.

terest rate policy (e.g., ensure $x_t = 0$), and simultaneously manipulate the path of the exchange rate via sterilized interventions in the currency market (by means of f_t^* in (13)). This result does not contradict the trilemma: FX interventions have real effects because market segmentation limits capital mobility and does not allow households to undo the open market operations of the central bank. As a result, the policymaker can move exchange rate risk between balance sheets of arbitrageurs and households, and thus change the equilibrium outcome in the currency market (*cf.* Wallace 1981, Silva 2016, Kekre and Lenel 2022).[21] Similarly to how nominal rigidities allow monetary policy to affect real outcomes, intermediation frictions give rise to an additional policy instrument f_t^* in (13). Note also that FX reserves are not essential for interventions as the same outcomes can be achieved using FX derivatives to absorb risk from the balance sheet of market participants. This result is consistent with a wide use of instruments such as currency swaps in central bank interventions (Patel and Cavallino 2019).

By this logic, the central bank can peg the nominal exchange rate in two different ways — using either monetary policy or FX interventions, affecting e_t in (12) by means of x_t and z_t, repsectively. The policy problem (15) identifies the costs and benefits associated with each of these implementations. On the one hand, monetary policy has the advantage that there are no restrictions on the implementable paths of the exchange rate — however, this comes at a cost of the output gap x_t. Unless prices are fully flexible, a monetary peg drives a wedge between the real exchange rate and its natural level $\tilde{q}_t$, resulting in suboptimal expenditure switching in the goods market (*cf.* "divine coincidence" below).

On the other hand, FX interventions can be used to manipulate the path of the exchange rate without any output gap side effects. However, there are important limits on the possible paths of the exchange rate that can be implemented with FXI. First, for a given monetary policy, FX interventions affect the nominal exchange rate by changing the real exchange rate, net exports and net foreign asset dynamics, via z_t in (11)–(13). Therefore, while FXI can temporarily alter the dynamics of the exchange rate, it is, for example, impossible to use them to generate a permanent appreciation, as it would result in a permanent trade deficit.[22] Second, FXI become entirely ineffective when monetary policy fully (and credibly) stabilizes the nominal exchange rate, $e_t = \bar{e}$ and hence $\bar{\sigma}_t^2 = 0$ in (13), bringing back the classic trilemma constraint (corresponding to the Peg point in Figure 1). This is the case because the currency supply by arbitrageurs becomes perfectly elastic in the absence of exchange rate risk, and they fully neutralize the effects of open market operations on the exchange rate. A continuous version of this result is that ever-increasing FX interventions are necessary to affect the exchange rate as currency demand becomes more elastic ($\bar{\omega}\bar{\sigma}_t^2 \to 0$).

Optimal monetary policy We introduce here a general characterization of optimal monetary policy for any given path of FX interventions, which nests as special cases the specific results that we consider

[21] It is the presence of a non-zero price of currency-holding risk, $\bar{\omega}\bar{\sigma}_t^2 > 0$ in (13), that relaxes the trilemma constraint on FXI and allows FXI to affect the equilibrium currency risk premium and thus the exchange rate.

[22] What happens when a policymaker attempts to fix the exchange rate at a level stronger than what is consistent with a long-run steady state, that is $\bar{e} < \tilde{q}$ in (12)? Statically, it can either result in a negative output gap, $x < 0$, or a positive risk-sharing wedge, $z > 0$ (i.e., excess tradable consumption). However, the latter is inconsistent with the intertemporal budget constraint (11) with NFA b_t^* exploding to negative infinity. Therefore, unless the monetary authority permits a negative output gap, an equilibrium with $\bar{e} < \tilde{q}$ and $z > 0$ can be sustained only temporarily, until a run on the government FX reserves (*cf.* Krugman 1979). Since targeting $z > 0$ is not part of an optimal policy profile, we do not explore this case further.

in turn in Sections 3.1–3.3. We prove in Appendix A3 that the solution to the policy problem (15) involves the following optimality condition:

Theorem 1 *For any given path of FX interventions $\{f_t^*\}$, the Ramsey optimal monetary policy sets the path of the output gap to satisfy $\mathbb{E}_t x_{t+1} = 0$ and:*

$$x_{t+1} = -\delta_t \cdot (e_{t+1} - \mathbb{E}_t e_{t+1}) \qquad \textit{with} \qquad \delta_t = \frac{2\gamma\bar{\omega}}{1-\gamma}\mu_t(n_t^* + f_t^* - b_t^*), \tag{16}$$

where δ_t is the intensity of the monetary policy lean against exchange rate surprises, $e_{t+1} - \mathbb{E}_t e_{t+1}$, and μ_t is the Lagrange multiplier on the risk-sharing constraint (13).

The optimality condition (16) connects the optimal path of monetary policy, summarized by the path of the output gap $\{x_{t+1}\}$, with three properties of the exchange rate and the currency market:

(i) exchange rate surprises, $e_{t+1} - \mathbb{E}_t e_{t+1}$;

(ii) capital outflows, or currency demand, $n_t^* + f_t^* - b_t^*$;[23]

(iii) departures from the first-best risk sharing and UIP, $\mathbb{E}_t \Delta z_{t+1} \neq 0$, as captured by the sequence of respective Lagrange multipliers μ_t on the risk-sharing constraint (13).

It is the interaction of these three features that determines the optimal monetary policy response, emphasizing already the role of non-linearity in the optimal exchange rate analysis captured by our approximation approach.

The policy lean δ_t in (16) corresponds to a free float when $\delta_t = 0$, a partial peg (or a managed float) when $\delta_t > 0$, or a full peg in the limit of $\delta_t \to \infty$.[24] In Figure 1, as δ_t increases from 0 to ∞, the equilibrium outcomes trace the entire red policy frontier from Free Float to Peg. A free float is optimal in the limit of a closed economy or a frictionless financial market. Our focus is on an open economy ($\gamma > 0$) with a frictional financial intermediation ($\bar{\omega} > 0$) where, in general, according to (16), optimal monetary policy responds to exchange rate surprises and, hence, deviates from the exclusive inward-looking goal of inflation and output gap stabilization ($x_{t+1} \equiv 0$). Specifically, the output gap in each period is eliminated on average, $\mathbb{E}_t x_{t+1} = 0$, but generally not state-by-state. In what follows, we first focus on two cases where the output gap is fully stabilized, $x_{t+1} \equiv 0$, either because capital outflows are fully accommodated with FXI, or when a fixed exchange rate is optimal by "divine coincidence". Then we consider the general case that can be described as the optimal crawling peg (or a dirty float), whereby optimal monetary policy compromises full output gap stabilization to smooth out exchange rate surprises.

3.1 Unconstrained optimal policy

When both policy instruments — monetary policy that controls the path of x_t and FXI f_t^* — are available and unconstrained, the first-best allocation is feasible and, thus, is implemented by the optimal policy.

[23]Note that a (gross) currency demand shock $n_t^* > 0$, unaccommodated with FXI, results in a (net) capital outflow, $z_t < 0$, at least when $\bar{\omega}\bar{\sigma}_t^2 > 0$. Therefore, we occasionally refer to n_t^* as capital outflow shocks.

[24]Using (12), we can rewrite (16) as $x_{t+1} = -\frac{\delta_t}{1+\delta_t}\big[\tilde{q}_{t+1} - z_{t+1} - \mathbb{E}_t(\tilde{q}_{t+1} - z_{t+1})\big]$, and in the limit of a full peg, x_{t+1} offsets one-for-one all exchange rate surprises that arise from $\tilde{q}_{t+1}$ and z_{t+1}.

Indeed, this corresponds to the special case of Theorem 1 where FXI ensure $n_t^* + f_t^* - b_t^* = 0$ in (16), which both results in $z_t = b_t^* = 0$ from (11)–(13) and renders optimal an inward-looking monetary policy $\delta_t = 0$ that eliminates the output gap $x_t = 0$.[25]

Proposition 1 *If both policy instruments are available and unconstrained, the optimal policy fully eliminates both wedges, the output gap $x_t = 0$ and the risk-sharing wedge $z_t = 0$, by targeting the output gap with monetary policy ($\delta_t = 0$) and demand for currency with FX interventions ($f_t = -n_t^*$). The nominal exchange rate varies with the natural real exchange rate, $e_t = \tilde{q}_t$, with conditional volatility $\bar{\sigma}_t^2 = \mathrm{var}_t(\tilde{q}_{t+1})$. This solution is unique, time consistent, and its implementation requires no commitment.*

Although the fact that two policy instruments are sufficient to implement the first-best allocation in the presence of two frictions is perhaps not surprising, the proposition shows that there is a one-to-one mapping between instruments and *optimal targets* (*cf.* Mundell 1962).[26] In particular, monetary policy closes the output gap $x_t = 0$ and stabilizes producer prices, while optimal FX interventions eliminate frictional UIP deviations, $i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = \mathbb{E}_t\Delta z_{t+1} = 0$, and thus close the risk-sharing wedge $z_t = 0$. Crucially, neither policy instrument targets the exchange rate directly, nor fully stabilizes it. Instead, optimal policy ensures $x_t = z_t = 0$, which in turn implies that the nominal exchange rate tracks the natural real exchange rate, $e_t = \tilde{q}_t$, and hence generally $\bar{\sigma}_t^2 = \mathrm{var}_t(\Delta\tilde{q}_{t+1}) > 0$. In Figure 1, this corresponds to the First Best, or a Friedman float, with inward-looking monetary policy ($\sigma_x = 0$) and with financial volatility (the red region) eliminated with FXI ($\sigma_e = \sigma_{\tilde{q}}$).

The proposition also provides a complementary characterization of the optimal policy in terms of responses to different types of shocks. Using the language of CGG, FX interventions *offset* currency demand shocks $f_t^* = b_t^* - n_t^* = -n_t^*$, as $b_t^* = 0$ in the first best, while allowing the exchange rate to *accommodate* fundamental macroeconomic shocks $\{A_t, Y_{Tt}, R_t^*\}$ that drive the natural real exchange rate $\tilde{q}_t$. To the extent financial intermediation is frictional and results in risk-sharing wedges, FX interventions should step in to eliminate the associated UIP deviations. In practice, this means providing FX liquidity to the market to offset currency demand shocks, eliminating the need for costly intermediation by absorbing the exchange rate risk exposure from arbitrageurs' and into the government balance sheet — a version of the Friedman (1969) rule.[27] The fact that interventions offset liquidity shocks state-by-state and are independent of expectations about future outcomes explains why the optimal policy is time consistent and does not require commitment on the part of the government.

An important feature of this setup is that it allows us to distinguish between UIP and CIP deviations, and to show that optimal policy should target the former. This contrasts with the conclusions of the previous literature where the limits to arbitrage arise due to financially-constrained arbitrageurs and CIP wedges are the only source of UIP deviations (Fanelli and Straub 2021, IPF). This difference is important

[25]Proposition 1, as well as Proposition 2 below, holds without approximation in the exact policy problem, as we show in Appendix A1. This is because the exact first-best allocation is feasible when the two policy instruments are available and unconstrained; similarly, it is feasible with a single monetary policy tool under the "divine coincidence" introduced below.

[26]Another notable feature of this result is that capital controls are not needed for implementation, as FX interventions are sufficient to achieve the first best allocation when combined with the optimal monetary policy (see Section 4.1).

[27]This result relies on two assumptions, namely the lack of opportunity costs of reserves (as they earn the market rate of return R_t^*) and no advantage of the financial sector in intermediation of capital flows relative to the policymaker. In the remainder of the analysis, we relax the first assumption by introducing binding constraints on the path of reserves, $f_t^* \in \mathcal{F}_t$.

from a practical perspective given that, in the data, UIP deviations are an order of magnitude larger than CIP deviations. More generally, FX interventions should be used to eliminate all rents in the currency market due to intermediation frictions, including the monopoly power of intermediaries. Because the policymaker is the agent on behalf of the households who cannot directly participate in FX trading, the portion of UIP deviations due to risk that is priced by the households (i.e., with their SDF) should not be eliminated with FXI. Consistent with Friedman (1953), the policymaker should take positions in the currency market as long as they are deemed profitable from the point of view of the households. As a result, the central bank should be making money, at least on average, from its FXI activity.

Implementation The optimal policy can be implemented using a conventional Taylor interest rate rule targeting inflation and the output gap, and a similar policy rule for FXI targeting ex ante UIP deviations. For example, adopting a rule $f_t^* = -\alpha \mathbb{E}_t \Delta z_{t+1}$ results in $\mathbb{E}_t \Delta z_{t+1} = \frac{\bar{\omega}\bar{\sigma}_t^2}{1+\alpha\bar{\omega}\bar{\sigma}_t^2}(n_t^* - b_t^*)$ from (13), which converges to $\mathbb{E}_t \Delta z_{t+1} = 0$ as $\alpha \to \infty$, and $f_t^* = b_t^* - n_t^*$ in this limit. In other words, FX interventions should lean against the wind intensively enough until the UIP wedge is entirely eliminated.

Despite its simplicity, the optimal FX policy might be hard to implement in practice. The challenge is that neither the UIP wedge $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$, nor liquidity shocks n_t^*, nor the natural level of the real exchange rate $\tilde{q}_t$ are directly observable in the data. One possibility is to condition the policy rule on observables that proxy for the policy target, e.g. the ex-post carry trade return $i_{t-1} - i_{t-1}^* - \Delta e_t$ or the level of the exchange rate, $f_t^* = -\alpha(e_t - \bar{e})$. In this case, the first-best implementation is generally infeasible, yet a policy that approaches a peg can be approximately optimal when financial shocks n_t^* dominate the volatility of the exchange rate relative to fundamental shocks $\tilde{q}_t$. The challenge of unobservable targets and shocks is, of course, not unique to FXI, as it is a common feature of optimal monetary policy in a closed economy, where the policymaker needs to make judgement calls about the natural rate of interest, potential output and NAIRU to offset shocks to aggregate demand and accommodate productivity shocks (see CGG). Even though not directly observable in the data, these concepts are useful in guiding the decisions of policymakers.

3.2 Divine coincidence

We consider now a special case of Theorem 1, whereby it is possible to achieve both policy objectives — in the goods and in the financial market — with a single monetary instrument, without recurring to capital flow or exchange rate management using FXI. By analogy with the closed-economy New Keynesian literature, we refer to this case as *divine coincidence*, and we further show in Section 5.1 how it generalizes the closed economy case.

The open-economy divine coincidence obtains when the first-best (natural) real exchange rate is stable at some level, $\tilde{q}_t = \bar{q}$. In this case, allowing for an arbitrary path of FXI $\{f_t^*\}$, a monetary policy rule that targets the same level of the nominal exchange rate, $e_t = \bar{q}$, both ensures a zero output gap and eliminates the risk-sharing wedge, $x_t = z_t = 0$, delivering the first best outcome. Indeed, in this case, $\bar{\sigma}_t^2 = \mathrm{var}_t(\Delta e_{t+1}) = 0$, and thus $z_t = 0$ is the unique solution of (11) and (13) independently of the path of (n_t^*, f_t^*). Given $z_t = 0$ and the fact that $\tilde{q}_t = \bar{q}$, expenditure allocation in the goods market (12) eliminates the output gap, $x_t = 0$, as the unique equilibrium outcome. We summarize this result in:

Proposition 2 *If the natural real exchange rate is stable, $\tilde{q}_t = \bar{q}$, then monetary policy that fully stabilizes the nominal exchange rate, $e_t = \bar{q}$, ensures the first best allocation with $x_t = z_t = 0$, for any path of FX interventions, including $f_t^* = 0$. An exchange rate peg is superior to inflation and output gap targeting, as it rules out multiplicity of exchange rate equilibria.*

In general, our model emphasizes the tension between the need for exchange rate adjustment in the goods market with sticky prices and the risk-sharing consequences of a volatile exchange rate unaccommodated by FXI. This dual role of the exchange rate, generally, makes a single policy instrument insufficient to attain efficiency in both goods and financial markets at once, as suggested by Theorem 1. Divine coincidence is the situation when this policy tradeoff disappears, as the natural real exchange rate is stable and, thus, a fixed nominal exchange rate does not compromise efficiency in the goods market. In turn, a nominal peg leads to a more elastic currency supply in the financial market and, in the limit, entirely frictionless intermediation. Thus, a fixed nominal exchange rate comes at no cost from the perspective of the goods market and delivers the first-best risk sharing from the perspective of the financial market. In fact, the fixed exchange rate policy is implied by Theorem 1, as $x_{t+1} = 0$ is consistent with $e_{t+1} = \mathbb{E}_t e_{t+1} = \bar{q}$ in this case, making sure the optimality condition (16) holds under a monetary peg, $\delta_t \to \infty$. In Figure 1, Divine (coincidence) corresponds to the case when $\sigma_{\tilde{q}} = 0$ and, hence, the blue region collapses to the origin with the Peg coinciding with the First Best.

Divine coincidence provides a rationale for pegging the exchange rate. Moreover, in this case, a nominal exchange rate peg by means of monetary policy is not only efficient, but also effective, as it eliminates the possibility of multiple equilibria. Consider the alternative policy of output gap (inflation) targeting that ensures $x_t = 0$ independently of the path of z_t. Under divine coincidence, such policy is consistent with an equilibrium with $e_t = \bar{q}$ and $z_t = \bar{\sigma}_t^2 = 0$. However, this is not a unique equilibrium, as there exists another equilibrium with arbitrageurs uncertain about the future exchange rate, $\bar{\sigma}_t^2 > 0$, which makes them charge a risk premium in response to currency demand shocks n_t^*, resulting in a self-fulfilling volatile exchange rate equilibrium.[28] The positive volatility equilibrium is suboptimal as it features $\mathbb{E}_0 z_t^2 > 0$ in contrast to the first best with $z_t = 0$. Thus, under divine coincidence, an exchange rate peg dominates inflation targeting, even though the result of the peg is zero inflation and a zero output gap (*cf.* Marcet and Nicolini 2003, Atkeson, Chari, and Kehoe 2010, Bianchi and Coulibaly 2023).

How special is the open-economy divine coincidence? On the one hand, this result extends immediately to various generalizations of the goods market with expenditure switching between varieties of home and foreign tradable goods (see Section 5). In each such model, one can define a concept of the natural real exchange rate that delivers efficient expenditure switching. A stable natural real exchange rate implies that a fixed nominal exchange rate does not come into conflict with the objectives of inflation and output gap stabilization in the goods market. At the same time, a stable natural real exchange rate is, of course, a knife-edge case which we do not expect to systematically hold in practice, yet it provides a useful benchmark and a stark illustration of the model's mechanism.

[28] If n_t^* follows an AR(1), then $e_t = \bar{q} - z_t$ follows an ARMA(2,1) with innovations proportional to the innovation of $\bar{\omega}\bar{\sigma}^2 n_t^*$, where $\bar{\sigma}^2 = \text{var}_t(e_{t+1}) > 0$ is a fixed point, in addition to the other fixed point with $\bar{\sigma}^2 = 0$. Note the difference of this multiplicity from the indeterminacy in Kareken and Wallace (1981).

On the other hand, the divine coincidence result is specific to the structure of the financial market that we assume in our framework. In particular, an ex post stable exchange rate, $e_{t+1} \equiv 0$, implies ex ante certainty, $\bar{\sigma}_t^2 = \mathrm{var}_t(e_{t+1}) = 0$, and this in turn guarantees that UIP holds and risk sharing is undistorted. Naturally, this requires that a peg is ex ante credible. Furthermore, this result relies on the structure of the model in which a fully stabilized exchange rate eliminates UIP deviations via the endogenous response of arbitrageurs who are willing to supply currency with infinite elasticity in the absence of exchange rate risk. If UIP deviations coexist with $\bar{\sigma}_t^2 = 0$, then the divine coincidence result breaks down. For example, this is the case when risk-sharing frictions are driven by balance sheet constraints rather than risk, and UIP and CIP deviations are closely linked. To the extent a credible peg eliminates a large portion of UIP deviations which are larger than CIP deviations — as the data seem to suggest (Itskhoki and Mukhin 2021b) — this result offers a useful quantitative benchmark.[29]

Optimal currency areas The divine coincidence result also provides an important benchmark for common currency areas, which are optimal when the natural real exchange rate between member countries is stable. In particular, this is the case when member countries share correlated fundamental shocks confirming the logic of Mundell (1961). What is new to our result is that it not only identifies the cases when the goods-market costs of a fixed exchange rate are low, but it also emphasizes the benefits of a fixed exchange rate from the perspective of the financial market. These benefits include reduced financial volatility and improved risk sharing between member countries. The benefits are larger the more the member countries trade with each other, as captured by the openness weight γ in the welfare loss function (10). Furthermore, we expect a fixed exchange rate — or a formation of a currency union — to dominate the alternative of an unmanaged (free) float, when the volatility of the bilateral nominal exchange rate under the float is dominated by non-fundamental currency demand shocks n_t^* relative to fundamental macro-trade shocks $\tilde{q}_t$.[30]

3.3 Crawling peg

Proposition 1 suggests that it is generally optimal to combine conventional monetary policy with FX interventions. However, in practice, it is not uncommon for countries to abstain from using FXI. This may be due to incomplete information about shocks and optimal targets in the currency market or due to additional constraints on the central bank's balance sheet, $f_t^* \in \mathcal{F}_t$, making it costly to intervene when FX reserves are too low (e.g., $f_t^* \geq 0$, non-negative reserves) or too high (e.g., $\bar{\sigma}_t |f_t^*| \leq \bar{\alpha}$, value-at-risk). In both cases, the central bank is prone to negative valuation effects, which can undermine its

[29]The red policy frontier in Figure 1 illustrates this property of the model. Unlike in the Trilemma models (blue frontier), our model features financial exchange rate volatility (z_t) in excess of fundamental volatility ($\tilde{q}_t$) under a float, yet it behaves like a trilemma model in the limit of a peg. Unlike in the models with exogenous transmission of financial shocks (yellow frontier), which do not feature divine coincidence, our model does not pass-on financial volatility (n_t^*) into the output gap under a peg, which allows the model to accommodate the Mussa facts on the change in exchange rate regimes.

[30]This insight is consistent with the experience of the Euro Zone, where the cost of borrowing was harmonized across countries and the cross-country financial flows increased significantly since the introduction of the euro in 1999 (Blanchard and Giavazzi 2002). Of course, an alternative interpretation is that these capital flows were excessive and driven by inefficient risk pricing of borrowing in Southern Europe, a case that may also arise in our model environment augmented with a possibility of default on net foreign liabilities resulting in fickle capital flows (Fornaro 2022).

credibility and lead to a loss of independence. Thus, we now study general implications of Theorem 1 for optimal monetary and exchange rate policy away from the first best, when FX interventions follow an arbitrary given path $\{f_t^*\}$, including $f_t^* = 0$ as one possibility.

Discretionary monetary interventions Before turning to the discussion of Ramsey-optimal policy, we first consider briefly the effects of discretionary ex post monetary interventions to stabilize the exchange rate and capital flows. We find that, without commitment, the optimal policy is always inward looking and focuses exclusively on the output gap. This is the case, even though, according to Theorem 1, an inward-looking monetary policy is generally suboptimal, provided there are departures from the first-best risk sharing. Ex post discretionary monetary interventions are allocative in the goods market and, in particular, affect the exchange rate. However, they cannot improve the allocation in the financial market — that is, they neither prevent capital outflows, nor improve international risk sharing or eliminate UIP wedges.

Proposition 3 *Without commitment, the optimal discretionary monetary policy stabilizes the output gap, $x_t = 0$. Discretionary ex post interventions that depart from $x_t = 0$, affect the exchange rate e_t, but do not change capital flows, UIP deviations or the risk-sharing wedge z_t.*

Proposition 3 shows that, in general, optimal discretionary monetary policy should not respond to the exchange rate, capital flows, or risk-sharing wedges. To see the intuition, consider an ex post monetary tightening and an associated reduction in output x_t carried out in response to a capital outflow shock — namely, an increase in currency demand n_t^* in (13) resulting in $z_t < 0$ and an exchange rate depreciation. Monetary tightening with $x_t < 0$ leads to an appreciation of the exchange rate e_t, offsetting the effect of $z_t < 0$, and expenditure switching away from home non-tradables in the goods market, according to the equilibrium condition (12).

This might be mistaken for a policy success to fend off the capital outflow shock. However, this outcome results only in costs in the goods market ($x_t < 0$) and no benefits in the financial market (as $z_t < 0$ still). Indeed, the equilibrium in the financial market, and in particular the path of the risk-sharing wedge z_t, is characterized by (11) and (13), which remain unaffected by a discretionary monetary tightening. Neither the size of the capital outflow, $n_t^* + f_t^* - b_t^*$, nor the unit price of risk $\bar{\omega}\bar{\sigma}_t^2$ in (13) respond to ex post monetary tightening. Discretionary policy affects the path of e_t and $\mathbb{E}_t \Delta e_{t+1}$, but it has no affect on $e_{t+1} - \mathbb{E}_t e_{t+1}$, and thus leaves the expected conditional volatility of the exchange rate, $\bar{\sigma}_t^2$, unchanged. As a result, the size of the UIP deviation (14) at t is also unaffected, emphasizing the futility of discretionary monetary interventions to manage capital flows.[31]

Commitment to a crawling peg We now return to Theorem 1, and consider the case when FXI do not ensure the first-best risk sharing ($n_t^* + f_t^* - b_t^* \neq 0$) and divine coincidence does not apply ($\tilde{q}_t \neq \bar{q}$),

[31]The result that monetary policy has no effect on capital flows whatsoever relies on the assumption that preferences are separable in tradables and non-tradables and no foreign intermediates are used in production. Despite being a special case, this provides a benchmark that illustrates the limited capacity of conventional monetary policy in capital flow management. Note that this result also generalizes to the case where noise shocks n_t^* are partially elastic to the expected UIP deviation which we show remains unchanged.

but the monetary authority can commit to a policy rule to respond to exchange rate surprises conditional on the state of the economy. In other words, we step outside of the special cases considered in Propositions 1–3, and study the general implications of the Ramsey-optimal monetary policy in an open economy, summarized in (16).

For simplicity, we consider a one-time deviation from the first-best FXI, and show that both a pure peg ($e_t = \bar{e}$) and a pure float (corresponding to $x_t = 0$) are, in general, suboptimal. Instead, the optimal monetary policy has a structure of a managed float, or a crawling peg, with $\delta_t \in (0, \infty)$ in (16).[32] That is, the policymaker commits to respond with monetary interventions x_t to smooth out surprise changes in the exchange rate, $e_t - \mathbb{E}_{t-1}e_t$, and in particular tighten monetary policy to accommodate depreciation shocks. We prove in Appendix A3:

Proposition 4 *Consider a case in which FXI $\{f^*_t\}$ are unconstrained in every period but t. Then $x_t = 0$ in every period except $t+1$, where:*

$$x_{t+1} = -\frac{2\gamma\bar{\omega}}{1-\gamma}\frac{\bar{\omega}\bar{\sigma}_t^2}{1+\beta+\bar{\omega}\bar{\sigma}_t^2}(n^*_t + f^*_t - b^*_t)^2(e_{t+1} - \mathbb{E}_t e_{t+1}), \tag{17}$$

which corresponds to the general optimality (16) with the Lagrange multiplier on the risk-sharing constraint (13) proportional to the UIP deviation, $\mu_t = (1+\beta+\bar{\omega}^2\bar{\sigma}_t^2)^{-1}\mathbb{E}_t\Delta z_{t+1}$.

To see the intuition, consider a state of the world with non-zero intermediated capital flows and a binding risk-sharing condition (13), so that $\mu_t(n^*_t + b^*_t - f^*_t) \neq 0$. As discussed above, adjusting monetary policy in period t does not affect contemporaneous capital flows. Instead, the policymaker can only indirectly mitigate the risk-sharing wedge by encouraging arbitrageurs to take larger positions and lowering the required risk premium. Monetary policy achieves this by leaning against surprise exchange rate innovations at $t+1$ and lowering the perceived conditional variances of the exchange rate, $\bar{\sigma}_t^2 = \mathrm{var}_t(\Delta e_{t+1})$. This makes financial intermediation less risky and relaxes the risk-sharing constraint (13). In particular, this implies that an unexpected depreciation, $e_{t+1} > \mathbb{E}_t e_{t+1}$, requires a monetary tightening that results in an output gap, $x_{t+1} < 0$. Importantly, this commitment does not depend on the source of volatility in the exchange rate at $t+1$ — namely, whether exchange rate surprises are driven by financial noise shocks n^*_{t+1} or fundamental macro shocks $\tilde{q}_{t+1}$.[33] Thus, optimal monetary policy is no longer inward-looking and limits the free float of the exchange rate.

Proposition 4 has several important implications. First, the optimal monetary policy always stabilizes the expected output gap, $\mathbb{E}_t x_{t+1} = 0$, irrespective of the path of the exchange rate e_t and the risk-sharing wedge z_t. Symmetrically, any expected change in the exchange rate, $\mathbb{E}_t\Delta e_{t+1}$, does not require accommodation with a monetary policy response. In other words, it is only exchange rate surprises, $e_{t+1} - \mathbb{E}_t e_{t+1}$, that require a policy response. Therefore, the optimal policy rule has the structure of a

[32] In Figure 1, this corresponds to the Managed Float point on the red policy frontier, describing the optimal compromise between exchange rate and output gap volatility, after the use of FXI to reduce the red area (financial volatility) has been exhausted given the policymaker's constraints.

[33] Substituting (16) into (12) and using $\mathbb{E}_t x_{t+1} = 0$ yields $e_{t+1} - \mathbb{E}_t e_{t+1} = \frac{1}{1+\delta_t}\big[(\tilde{q}_{t+1} - \mathbb{E}_t\tilde{q}_{t+1}) - (z_{t+1} - \mathbb{E}_t z_{t+1})\big]$, which shows how the optimal policy lean $\delta_t > 0$ dampens equally the exchange rate surprises from $\tilde{q}_{t+1}$ and z_{t+1}.

crawling peg (band) — it fully allows for expected exchange rate adjustment and responds only to unexpected exchange rate movements. The implication is that any medium-run exchange rate adjustment can be accommodated with expected exchange rate changes — a *managed float* or a *crawling band* — limiting the trade-off to be exclusively about managing high-frequency exchange rate movements.[34]

Second, optimal monetary interventions in the currency market are state-contingent. Taking advantage of non-linearity allowed by our approximation, Proposition 4 shows that the intensity of the optimal policy lean δ_t increases with the size of the frictional UIP wedge (13) and the size of the capital (out)flow shock. A constant-intensity policy rule, $\delta_t \equiv \delta$, is feasible, and results in a constant conditional exchange rate volatility, $\bar{\sigma}_t^2 = \bar{\sigma}^2$.[35] However, it is suboptimal and dominated by a state-contingent policy rule with a policy lean:

$$\delta_t = \frac{2\gamma\bar{\omega}}{1-\gamma}\frac{\bar{\omega}\bar{\sigma}_t^2}{1+\beta+\bar{\omega}\bar{\sigma}_t^2}(n_t^* + f_t^* - b_t^*)^2,$$

which is both increasing in the unit price of the exchange rate risk, $\bar{\omega}\bar{\sigma}_t^2$, and increasing and convex in the size of unaccommodated capital outflow shocks, $|n_t^* + f_t^* - b_t^*|$. Recall from (13)–(14) that the size of the frictional UIP deviation is given by $\mathbb{E}_t\Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*)$, and thus the optimal policy lean is quadratic in the UIP deviation.

It follows that the crawling peg is more relevant for countries with a larger tradable sector γ and, thus, higher welfare costs of capital flow shocks. Furthermore, periods with larger expected exchange rate volatility, $\bar{\sigma}_t^2$, and larger excess demand for currency, $|n_t^* + f_t^* - b_t^*|$, call for a commitment to a stronger future response of monetary policy x_{t+1} to exchange rate surprises $e_{t+1} - \mathbb{E}_t e_{t+1}$. This suggests a state-contingent policy approach to financial market volatility, which can be ignored when it causes no spikes in risk premia (intermediation wedges), but should be smoothed out with increasing intensity using monetary policy tools when financial volatility distorts risk sharing and direct financial market interventions (FXI) are constrained.

3.4 Optimal FXI and forward guidance

We have focused so far on cases when the first-best allocation is implementable in all but perhaps one period, which significantly simplifies the analysis and allows us to solve for the optimal policy rule in Proposition 4. More generally, Theorem 1 shows that optimal policy depends on the history of previous shocks as well as expectations about their future realizations as summarized by the endogenous Lagrange multipliers μ_t. While no closed-form solution is available in the general case, we provide here a further characterization of the second-best optimal policies.

[34] One policy option is to set a very narrow band which in the limit approximates $e_{t+1} = \mathbb{E}_t e_{t+1}$, where $\mathbb{E}_t e_{t+1}$ satisfies the intertemporal budget constraint given fundamental shocks up to time t. Such policy rule fully eliminates financial volatility (as $\bar{\sigma}_t^2 = 0$) at the cost of a delayed adjustment to fundamental shocks. Some oil exporting countries, such as Saudi Arabia, follow a comparable policy, in parallel accumulating an FX sovereign wealth fund for the future when global demand for oil declines. If the present value of all future oil revenues is predictable, this rule approximates the first-best policy.

[35] In the class of constant-lean policies, one can optimize over $(\delta, \bar{\sigma}^2)$ to show that δ is increasing in openness γ and in the ratio of financial-noise n_t^* to macro-fundamental $\tilde{q}_t$ volatility. See Kollmann (2004) for a quantitative analysis of this trade-off in a model with exogenous UIP deviations that are assumed to vanish under a fixed exchange rate.

When FXI are unconstrained in period t, the risk-sharing constraint (13) is not binding and $\mu_t = 0$. Theorem 1 then implies that monetary policy at $t+1$ can focus solely on closing the output gap, $x_{t+1} = \delta_t = 0$, irrespective of binding risk-sharing constraints in any other periods. However, even in this case, optimal FXI do *not* necessarily just eliminate the UIP deviation at t. The following result provides a general characterization of the optimal path of FX interventions and UIP deviations:[36]

Theorem 2 *For any path of monetary policy $\{x_t\}$, and with occasionally binding constraints on FX interventions $f_t^* \in \mathcal{F}_t$, the optimal UIP deviation at t is given by:*

$$\begin{aligned}\mathbb{E}_t\Delta z_{t+1} =& \big(1+\beta+\bar{\omega}\bar{\sigma}_t^2\big)\mu_t \\ &- \beta\mathbb{E}_t\mu_{t+1} - \big[1+2\bar{\omega}(n_{t-1}^*+f_{t-1}^*-b_{t-1}^*)(e_t-\mathbb{E}_{t-1}e_t)\big]\mu_{t-1},\end{aligned} \tag{18}$$

and it is supported by f_t^* that satisfies (13), that is $\bar{\omega}\bar{\sigma}_t^2(n_t^*+f_t^*-b_t^*) = \mathbb{E}_t\Delta z_{t+1}$.

First, note that optimal FXI focus exclusively on improving the allocation in the financial market and do not respond to the output gap x_t, even when monetary policy is constrained, as f_t^* cannot affect the path of x_t. Second, even when FXI are unconstrained at t, they do not necessarily offset currency demand shocks as in the first best. Indeed, UIP is optimally distorted, $\mathbb{E}_t\Delta z_{t+1} \neq 0$, if either $\mu_{t-1} \neq 0$ or $\mathbb{E}_t\mu_{t+1} \neq 0$ in (18). The latter effect is *macroprudential* FXI in anticipation of the future binding risk-sharing constraint at $t+1$. Conversely, μ_{t-1} captures *forward-guidance* FXI to alleviate the UIP deviation at $t-1$, which requires commitment.[37] Both forward-guidance and preemptive FXI require a larger intervention at t, that is $f_t^* < b_t^* - n_t^*$ when there is an unaccommodated capital outflow either at $t+1$ or at $t-1$. We illustrate these effects in Appendix Figure A2 using an example with a full analytical solution provided in Appendix A3.

Thus, an unconstrained use of FXI at t does not just eliminate UIP deviations at t, but also smoothes out UIP violations in previous and future periods. Nonetheless, this does not imply that the optimal policy in any given period depends on the entire sequence of past and future binding constraints. Perhaps surprisingly, we show next that the Ramsey policy features both optimal *amnesia* and *myopia*.

Proposition 5 (i) *If FXI are unconstrained at $t-1$, $\mu_{t-1} = 0$, then the optimal policy $\{\delta_{t+j}, f_{t+j}^*\}_{j\geq 0}$ does not depend on the previous history ("amnesia").* (ii) *If, in addition, $\mu_t = 0$, then the optimal policy at t, (δ_t, f_t^*), is the same under commitment and under discretion.* (iii) *If FXI are also expected to be unconstrained on average at $t+1$, $\mathbb{E}_t\mu_{t+1} = 0$, the optimal policy closes both the output gap and the UIP deviation, $\delta_t = \mathbb{E}_t\Delta z_{t+1} = 0$, irrespective of any future shocks and biding constraints ("myopia").*

Unlike with the output gap (16), it is only optimal to eliminate the period t UIP deviation if FXI are unconstrained simultaneously at $t-1$, t and $t+1$. Otherwise, risk sharing at t is distorted due to either

[36]The optimality condition (18) both determines the optimal UIP deviation given the path of Lagrange multipliers $\{\mu_t\}$ and characterizes the dynamics of μ_t given equilibrium UIP deviations. Conditions (16) and (18), together with constraints (11)–(13), characterize the Ramsey solution $\{x_t, f_t^*, z_t, b_t^*, e_t, \bar{\sigma}_t^2, \mu_t\}$ to problem (15) given constraints on the path of policies.

[37]Similarly to conventional monetary guidance, the policymaker exploits the fact that z_{t-1} is forward-looking and depends on $\mathbb{E}_{t-1}z_t$ in (13). In addition, and differently from conventional forward guidance, future FXI also lean against surprises, stabilizing z_t around $\mathbb{E}_{t-1}z_t$ to reduce the exchange rate volatility $\bar{\sigma}_{t-1}^2$. The two terms in the bracket in front of μ_{t-1} in (18) correspond to these two forward guidance channels, with the latter mirroring the optimal monetary policy lean δ_{t-1} in (16).

current, past or future shocks and a limited ability to offset them with FXI. However, no past shocks and frictions — summarized by $\{\mu_{t-j}\}_{j\geq 0}$ — matter at t if FXI is unconstrained at $t-1$ and, hence, $\mu_{t-1}=0$. Therefore, the optimal FXI exhibit memory loss after the first unconstrained state is reached. In that state, UIP is optimally distorted to absorb all intertemporal spillovers from the earlier risk-sharing wedges. Similarly, no future shocks and binding constraints, $\{\mu_{t+j}\}_{j>0}$, matter for the optimal intervention at t if FXI are unconstrained at $t+1$ and, hence, $\mu_{t+1}=0$. Unlike with monetary guidance, full FXI guidance is achieved in one unconstrained period because the use of reserves introduces no distortions (like output gaps) or costs that need to be smoothed over time.

Similar logic applies to future expected shocks and binding risk-sharing constraints, which trigger an early use of FXI and backward spillovers into anticipatory UIP deviations when current FX reserves are insufficient to fully offset or stop the propagation of the shock. Nevertheless, there is an important asymmetry between past and future shocks as only the former ones require commitment. Indeed, without commitment, the government cannot fulfill its past promises, and the optimal policy is generally not time consistent at t, unless $\mu_{t-1}=\mu_t=0$. In contrast, the optimal policy response to future shocks (that result in $\mu_{t+1}\neq 0$) is time consistent and includes both preemptive FXI and NFA accumulation (whether official f_t^* or private b_t^*) that alleviate expected future distortions.

4 Extensions

4.1 International transfers and capital controls

This section generalizes the baseline model to feature capital control taxes and international wealth transfers due to valuation effects on cross-border asset holdings. The goal of this analysis is twofold. First, we study whether capital controls can substitute for other policy instruments when the latter are constrained. Second, we explore the optimal policy mix in the presence of cross-border rents from international currency provision.

Towards these goals, we extend the financial sector to additionally feature foreign financial actors — both liquidity traders and intermediaries. Specifically, the aggregate liquidity demand for currency originates from both domestic and international noise traders, $N_t^* = N_{Ht}^* + N_{Ft}^*$. There are also domestic and foreign intermediaries — of measure m_H and m_F, respectively — that supply currency D_{Ht}^* and D_{Ft}^* according to a portfolio choice rule similar to (4). Appendix A4 contains a detailed description of the environment and derivations, while this section outlines the results.

We further allow for a rich set of taxes. In particular, we assume that domestic households face a tax τ_t^h on their home-currency deposits, so that the after-tax return on their asset position is $R_t/(1+\tau_t^h)$. Domestic financial agents — both noise traders and intermediaries — are subject to a pair of taxes (τ_{Ht},τ_{Ht}^*) on their home-currency and foreign-currency positions, respectively. As a result, their after-tax carry-trade return is given by $\tilde{R}_{Ht+1}^* = \frac{R_t^*}{1+\tau_{Ht}^*} - \frac{R_t}{1+\tau_{Ht}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$. In contrast, foreign financial agents face only a tax τ_{Ft} on their home-currency position, resulting in an after-tax carry-trade return $\tilde{R}_{Ft+1}^* = R_t^* - \frac{R_t}{1+\tau_{Ft}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$. No other asset holdings are in the domestic policymaker's tax jurisdiction, and in particular she cannot tax either foreign households or the foreign-currency positions of foreign traders (see Appendix Figure A3). Without loss of generality, we interpret (τ^*_{Ht}, τ_{Ft}) as ex-ante taxes on cross-border asset positions — that is, preemptive capital controls (Das, Gopinath, and Kalemli-Özcan 2022).

The presence of foreign financial agents results in an international wealth transfer from incomes and losses on their carry trade positions. Specifically, $\tilde{R}^*_{Ft+1} \cdot \frac{N^*_{Ft}+D^*_{Ft}}{R^*_t}$ is the income transfer from home to the rest of the world, and it is subtracted from the home country budget constraint (6).[38] Furthermore, asset taxes introduced above affect the equilibrium risk-sharing condition (7). We show in the appendix that the generalized risk-sharing condition with asset taxes can be written as:

$$\beta R^*_t \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = \frac{1+\tau^h_t}{1+\tau_t} + \frac{\omega\sigma^2_t}{(1+\tau_t)^2}\frac{B^*_t - N^*_t - F^*_t}{R^*_t}, \tag{19}$$

where τ_t can represent one of two capital control policies: (i) a common tax on home-currency asset holdings for both home and foreign financial agents, $\tau_{Ht} = \tau_{Ft} = \tau_t$, or (ii) a capital control tax on inflows and a subsidy on outflows, $\tau_{Ft} = \frac{-\tau^*_{Ht}}{1+\tau^*_{Ht}} = \tau_t$. Note that these capital control policies work regardless of the composition of home and foreign intermediaries and noise traders. The other equilibrium conditions remain unchanged.

The generalized risk-sharing condition (19) clarifies two important properties of capital controls. From the point of view of risk sharing, capital control taxes τ_t and quantity interventions in FX markets F^*_t are substitutes and can be used interchangeably to offset the effect of liquidity shocks N^*_t on macroeconomic allocations and risk sharing. In particular, there are three ways in which asset taxes can be used to offset the distortionary effect of a capital outflow shock, $N^*_t > 0$:

(i) with a savings tax on households, $\tau^h_t > 0$, which encourage tradable consumption despite depreciated exchange rate (see the analysis of financial repression in Itskhoki and Mukhin 2022);

(ii) a home-currency investment subsidy for an entire financial sector, $\tau_{Ht} = \tau_{Ft} = \tau_t < 0$, which allows the financial sector to collect carry trade returns and accommodate the currency demand shock without distorting the household risk-sharing condition;

(iii) a capital controls policy, taxing foreign-currency positions of domestic agents, $\tau^*_{Ht} = \frac{-\tau_t}{1+\tau_t} > 0$, and subsidizing home-currency positions of foreigners, $\tau_{Ft} = \tau_t < 0$, again resulting in a positive carry trade return for financial agents without distorting the household risk sharing.

The feature of all three policies is that they generate a wedge in returns between households and financial sector, decoupling carry trade returns from the risk-sharing wedge.[39]

In practice, however, the use of capital controls is complicated by the need to set state-contingent tax rates that vary significantly over time. Furthermore, the planner may be unable to distinguish between different types of agents and capital flows to impose agent- and asset-specific capital controls.

[38] Note that, while N^*_{Ft} is an exogenous asset position (liquidity shock), the position of intermediaries depends on the expected carry-trade return, $\frac{D^*_{Ft}}{R^*_t} = \frac{\mathbb{E}_t\{\Theta_{t+1}\tilde{R}^*_{Ft+1}\}}{\omega\sigma^2_t/(1+\tau_{Ft})^2}$. Thus, arbitrageurs invest in a portfoflio with a positive expected return and make positive expected profits. Therefore, rents can be extracted systematically only from positions against noise traders.

[39] Another approach involves taxing profits (and subsidizing losses) from financial transactions to effectively increase the risk-absorption capacity of intermediaries $(1/\omega)$, however, this requires modeling entry and exit of arbitrageurs (Atkeson, Eisfeldt, and Weill 2015). An alternative set of tax instruments involves import and export taxes, or so-called fiscal devaluations, that introduce a tax wedge in the goods market equilibrium condition (12) instead of (20), and can remove the goods market distortions associated with a fixed exchange rate (Farhi, Gopinath, and Itskhoki 2014).

For example, condition (19) illustrates that setting a uniform tax on home bonds for all agents, $\tau_t^h = \tau_{Ht} = \tau_{Ft} = \tau_t$, does not have the desired effect, as it is equivalent to a shift in the home-currency nominal interest rate R_t that cannot offset capital outflow shocks (recall Proposition 3). Finally, capital control policies generally require subsidies and are not budget-balanced, unlike optimal FXI that by construction generate revenues on average.

With these caveats in mind, we proceed with the analysis of optimal policies specializing to the case of capital controls on international inflows and outflows, $\tau_{Ft} = \frac{-\tau_{Ht}^*}{1+\tau_{Ht}^*} = \tau_t$, without the use of domestic asset taxes, $\tau_t^h = \tau_{Ht} = 0$. We follow the same approach as in Section 2.2 and approximate the policy problem around the planner's allocation, which now takes into account the possible rent extraction from foreign traders. We denote with $\psi_t \equiv i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} - \tau_t$ the expected carry trade return for both home and foreign investors, which also equals the after-tax UIP deviation.

We prove the following two results that characterize equilibrium risk-sharing and international transfers. First, the generalized risk-sharing condition (13) is now:

$$\mathbb{E}_t\Delta z_{t+1} = \psi_t + \tau_t, \qquad \text{where} \qquad \psi_t = \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*) \quad \text{and} \quad \bar{\sigma}_t^2 = \mathrm{var}_t(e_{t+1}). \tag{20}$$

Capital controls τ_t decouple the household risk-sharing wedge z_t dynamics from the risk premium ψ_t that intermediaries charge to accommodate international capital flows. In other words, while capital controls can substitute for FXI f_t^* to eliminate the risk-sharing wedge, they do so without eliminating the equilibrium risk premium (*cf.* Jeanne 2022). Similarly to the domestic interest rate R_t, changes in capital controls τ_t are absorbed by expected depreciation $\mathbb{E}_t\Delta e_{t+1}$ and do not affect the expected after-tex carry-trade returns ψ_t, which are determined by the balance of supply and demand in the currency market (20).

Second, the expected transfer of carry-trade incomes and losses to the rest of the world is given by $\beta\big(\frac{m_F}{\bar{\omega}\bar{\sigma}_t^2}\psi_t - n_{Ft}^*\big)\psi_t$, where m_F is the share of foreign intermediaries and $n_{Ft}^* = N_{Ft}^*/\bar{Y}_T$ is the foreign noise trader demand shock normalized by tradable output. This leads to the following second-order approximation to the welfare loss function around the planner's allocation:

$$\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[(1-\gamma)x_t^2 + \gamma z_t^2 + 2\beta\gamma\left(\frac{m_F}{\bar{\omega}\bar{\sigma}_t^2}\psi_t - n_{Ft}^*\right)\psi_t\right]. \tag{21}$$

The policymaker chooses the path of policies $\{x_t, f_t^*, \tau_t\}$ and equilibrium outcomes $\{z_t, e_t, b_t^*, \psi_t, \bar{\sigma}_t^2\}$ to minimize (21) subject to (20), as well as the original constraints (11) and (12). In addition to minimizing the loss from the output gap and the risk-sharing wedge, the objective now also includes minimizing transfers abroad to foreign traders.[40] Matching the three policy targets, the policymaker now has access to three instruments, which include capital controls τ_t in addition to monetary policy x_t and FXI f_t^*.

[40]Interestingly, to the second-order approximation, international rents depend only on expected returns, while ex-post valuation effects are of a higher order. As a result, the expression for transfers is largely isomorphic to the one in a deterministic case with CIP deviations replaced by expected UIP deviations (*cf.* Fanelli and Straub 2021). Furthermore, this implies that, given the structure of international asset markets and the order of approximation, the planner does not aim to use state-contingent valuation effects to 'complete the markets' (as in Fanelli 2017).

Proposition 6 (a) *An undistorted allocation without international transfers, $x_t = z_t = \psi_t = 0$, is always feasible and requires the use of FXI without capital controls.* (b) *When $n^*_{Ft} \neq 0$, the planner can increase welfare with an international transfer by only partially accommodating the foreign liquidity demand, $\psi_t = \frac{\bar{\omega}\bar{\sigma}_t^2}{2m_F} n^*_{Ft}$, and still ensuring $x_t = z_t = 0$ with capital controls, $\tau_t = -\psi_t$.* (c) *When $m_F > 0$, further gains are achieved by deviating from $x_t = z_t = 0$ to increase $\bar{\sigma}_t^2$ and the optimal transfer from abroad.*

The first part of the proposition emphasizes that the undistorted outcome of Proposition 1 is still feasible when parts or all of the financial sector is off-shore. Indeed, implementing the baseline first-best monetary and FXI policies, $x_t = 0$ and $f^*_t = -n^*_t$, still ensures the undistorted allocation and a zero international transfer, as this policy closes the UIP gap and, hence, eliminates expected carry trade returns, $\psi_t = 0$. Importantly, in doing so, capital controls cannot substitute for FXI, as they eliminate the risk-sharing wedge without eliminating the carry trade returns. Thus, in the presence of international financial actors, the side effect from the use of capital controls to eliminate the risk-sharing wedge is an international transfer. In general, this transfer may be positive or negative. However, in an important special case when all noise traders are domestic ($n^*_{Ft} = 0$) and there are some foreign intermediaries ($m_F > 0$), the expected transfer term in (21) is weakly negative, making $x_t = z_t = \psi_t = 0$ the best unconstrained policy outcome.[41]

In the presence of foreign liquidity demand for domestic currency — whether $n^*_{Ft} < 0$ or $n^*_{Ft} > 0$ — a country can generate rents from the rest of the world by exploiting the monopoly power it has in supplying currency. The second part of Proposition 6 shows that when $n^*_{Ft} \neq 0$, it is no longer optimal to accommodate the entire currency demand n^*_t with FX interventions, as in Proposition 1. Furthermore, it is possible to both eliminate wedges, $x_t = z_t = 0$, and ensure positive rents in the currency market. This requires using capital controls to eliminate the risk-sharing wedge in (20), $\tau_t = -\psi_t$, which ensures $z_t = 0$ regardless of the equilibrium UIP deviation ψ_t. The maximum government revenues from interventions are attained by ensuring that $\psi_t = \frac{\bar{\omega}\bar{\sigma}_t^2}{2m_F} n^*_{Ft}$, which is the peak of the rents term in (21). In turn, this requires that FXI fully satisfy the currency demand of domestic noise traders and only partially for foreign noise traders, e.g. $f^*_t = -n^*_{Ht} - \frac{1}{2} n^*_{Ft}$ when all intermediaries are foreign ($m_F = 1$).[42]

Collecting rents requires leaving positive carry trade returns on the table for the intermediaries, who in turn constrain the maximum rent extraction by the government. The elasticity of currency supply by foreign intermediaries is $\frac{m_F}{\bar{\omega}\bar{\sigma}_t^2}$, exactly as it appears in the wealth transfer term in (21) and in the expression for the rent-maximizing UIP deviation ψ_t. Unlike in the related analyses of Fanelli and Straub (2021) and IPF, this elasticity is endogenous to policy via the equilibrium exchange rate volatility, $\bar{\sigma}_t^2 = \mathrm{var}_t(e_{t+1})$. The last part of Proposition 6 shows that to maximize rents and the policy objective, the planner departs from $x_t = z_t = 0$ to reduce the elasticity of intermediary currency supply. Specifically, this requires amplifying the exchange rate shocks with both monetary policy deviations x_t and controls on capital flows z_t to increase the resulting exchange rate volatility $\bar{\sigma}_t^2$ above $\mathrm{var}_t(\tilde{q}_{t+1})$.[43]

[41] The literature often focuses on this case, where policy generates no expected rents and is used towards other objectives at the cost of international income loss (Jeanne 2012, Amador, Bianchi, Bocola, and Perri 2019, Fanelli and Straub 2021).

[42] In practice, the non-negative FX reserves constraint $f^*_t \geq 0$ makes it hard to collect rents when traders short home currency $n^*_{Ft} < 0$, explaining the observed asymmetry between Switzerland and Argentina.

[43] The proof in the appendix shows that the optimal deviations satisfy $(1-\gamma)x_t = \gamma \mathbb{E}_t \Delta z_{t+1} = \frac{\gamma\bar{\omega}}{2}(n^*_{Ft-1})^2(e_t - \mathbb{E}_{t-1}e_t)$, amplifying exchange rate surprises following foreign noise-trader shocks, in reverse to stabilization policy in (16)–(18).

To summarize, optimal FX interventions always lean against UIP deviations, but stop short of fully offsetting the foreign liquidity demand for currency. This leaves the UIP premium partially open to ensure positive equilibrium rents. Echoing the recent experience of Switzerland, a positive demand for home currency should be addressed by issuing reserves and accumulating assets in foreign currency, while simultaneously imposing capital controls or allowing the exchange rate to partially appreciate (Bacchetta, Benhima, and Berthold 2023).

4.2 International cooperation

Up until now, we have focused on optimal policy in a small open economy that takes as given global economic conditions, in particular the world interest rate. This section studies international spillovers and optimal policy coordination in a multi-country world. Towards this end, we consider a world comprised of a continuum of small open economies index by $i \in [0, 1]$, each one isomorphic to the country in our baseline model, with country $i = 0$ (the US, denoted with $*$) issuing the global funding currency (the dollar). We denote with $m_0 \geq 0$ the measure of countries $i \in (0, m_0]$ that form a dollar currency union (or dollar pegs), which in particular nests the case of a non-infinitesimal US economy when $m_0 > 0$. There is a global market for the tradable endowment good and a sticky-price non-tradable production sector in each economy, as in the baseline model. The law of one price still holds for tradables, and now we write it in logs as $p_{it} = p^*_{Tt} + e_{it}$ for all $i \in (0, 1]$ with e_{it} denoting the country i nominal exchange rate against the dollar. In general, we now allow $p^*_{Tt} \neq 0$, and $\pi^*_{Tt} \equiv \Delta p^*_{Tt}$ denotes the US dollar tradable inflation.

We make two assumptions about the structure of the asset market. First, only nominal dollar bonds are available for international risk sharing, which generates an asymmetry between the US and other economies. Second, for each currency there is a separate market, in which agents can trade it against the dollar. This segmentation of currency markets is in line with the fact that the dollar accounts for 88% of the global FX market turnover, but it is not crucial for our results which remain largely unchanged if one assumes that arbitrageurs can invest simultaneously in a portfolio of currencies. For simplicity, we assume local financial markets, as in the baseline model, to exclude the redistributive motive in national policies discussed in the previous subsection. Appendix A4 provides detailed derivations.

The equilibrium conditions for a given economy are the same as in the baseline model described in Section 2. Instead, the main difference is that the international real interest rate, $r^*_t = i^*_t - \mathbb{E}_t \pi^*_{Tt+1}$, is endogenous, and it is shaped by the global market clearing condition for tradables, $\int_0^1 c_{it} \mathrm{d}i = y_{Tt}$, where $y_{Tt} \equiv \int_0^1 y_{it} \mathrm{d}i$ is the aggregate global endowment of tradables at t. We show in the appendix that, to the first order, the global planner that cannot directly redistribute wealth across countries equalizes the expected consumption growth, $\mathbb{E}_t \Delta \hat{c}_{it+1} = \mathbb{E}_t \Delta y_{Tt+1}$ for all $i \in [0, 1]$, where $\hat{c}_{it}$ is country i tradable consumption in the global planner's allocation. Consequently, the real interest corresponding to the planners allocation equals the expected growth rate of the aggregate endowment, $\hat{r}^*_t = \mathbb{E}_t \Delta y_{Tt+1}$.[44] Recall that the local country i planner chooses $\mathbb{E}_t \Delta \tilde{c}_{it+1} = r^*_t$ taking r^*_t as given,

[44]Under sticky prices, the global planner's allocation is decentralized with the US monetary policy $i^*_t = -\mathbb{E}_t \Delta a_{0t+1}$, where a_{0t} is the log non-tradable productivity in the US, expected tradable price inflation $\mathbb{E}_t \pi^*_{Tt+1} = \mathbb{E}_t\{\Delta a_{0t+1} - \Delta y_{Tt+1}\}$, and nominal exchange rates $e_{it} = a_{it} - \hat{c}_{it} - p^*_{Tt}$ for $i \in (0, 1]$, where $p^*_{Tt} = a_{0t} - \hat{c}_{0t}$.

whether or not it is equal to $\hat{r}_t^*$, and therefore $\tilde{c}_{it}$ may differ from $\hat{c}_{it}$.

Away from the optimal allocation, we define the output gap x_{it} as before, while the global risk-sharing wedge is now $\hat{z}_{it} \equiv c_{it} - \hat{c}_{it}$, which differs from the local wedge $z_{it} = c_{it} - \tilde{c}_{it}$ when $r_t^* \neq \hat{r}_t^*$. Interestingly, as our next Lemma shows, the UIP deviation (14) still corresponds to the local wedge, $\psi_{it} \equiv \mathbb{E}_t \tilde{z}_{it+1}$, while the global wedge additionally reflects the aggregate interest rate wedge, $\bar{\psi}_t \equiv -(r_t^* - \hat{r}_t^*)$, where r_t^* is the equilibrium interest rate.[45]

Lemma 1 *In an equilibrium with sticky prices and frictional financial intermediation, the aggregate welfare loss relative to the global planner's allocation up to second order is given by:*

$$\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \left[\gamma \hat{z}_{it}^2 + (1-\gamma) x_{it}^2\right] \mathrm{d}i, \tag{22}$$

and the global risk-sharing wedges $\hat{z}_{it}$ satisfy market clearing $\int_0^1 \hat{z}_{it}\mathrm{d}i = 0$ and risk-sharing conditions:

$$\mathbb{E}_t \Delta \hat{z}_{it+1} = \psi_{it} - \bar{\psi}_t \qquad \textit{for all} \quad i \in [0,1], \tag{23}$$

where $\psi_{it} \equiv \bar{\omega}_i \bar{\sigma}_{it}^2 (n_{it}^* + f_{it}^* - b_{it}^*)$ with $\bar{\sigma}_{it}^2 \equiv \mathrm{var}_t(e_{it+1})$ is the currency i UIP wedge, and $\bar{\psi}_t \equiv \int_0^1 \psi_{it}\mathrm{d}i$ is the aggregate real interest rate wedge, $r_t^* - \hat{r}_t^* = -\bar{\psi}_t$.

The risk-sharing condition (23) is the generalization of (13) which takes into account the endogeneity of the world interest rate r_t^*. As before, the risk sharing for country i — and hence UIP for currency i — is distorted when excess demand for the dollar relative to currency i, $n_{it}^* + f_{it}^* - b_{it}^*$, needs to be absorbed by the intermediaries, provided that $\bar{\omega}_i \bar{\sigma}_{it}^2 \neq 0$. The risk-sharing condition for the US, $i = 0$, as well as for all countries that peg to the dollar, $i \in (0, m_0]$, is therefore $\mathbb{E}_t \Delta \hat{z}_{it+1} = -\bar{\psi}_t$ with UIP satisfied, $\psi_{it} = 0$.

In addition, (23) now features a global interest rate wedge $\bar{\psi}_t$ common for all countries, which arises due to unaccommodated shifts in the global demand for dollars, $\bar{n}_t^* \equiv \int_0^1 n_{it}^* \mathrm{d}i$. In particular, an increase in global dollar demand, $\bar{\psi}_t = \int_0^1 \bar{\omega}_i \bar{\sigma}_{it}^2 (n_{it}^* + f_{it}^* - b_{it}^*)\mathrm{d}i > 0$, depresses the world real interest rate below its efficient level $\hat{r}_t^*$. It also results in correlated UIP premia on non-dollar-pegged currencies, $i \in (m_0, 1]$, capital outflows from these countries, and depressed tradable consumption, $\hat{z}_{it} < 0$. The resulting *global savings glut* and the depressed world real interest rate r_t^* result in suboptimal capital inflows and, hence, current account deficits in the US and dollar-pegged countries, $\hat{z}_{it} > 0$ for $i \in [0, m_0]$ (*cf.* Caballero, Farhi, and Gourinchas 2008, Mendoza, Quadrini, and Ríos-Rull 2009).

A cooperative optimal policy minimizes the aggregate welfare loss (22) subject to market clearing, the risk-sharing conditions (23), as well as the expenditure switching conditions

$$e_{it} = \hat{q}_{it} - p_{Tt}^* + x_{it} - \hat{z}_{it}, \tag{24}$$

[45] The proof takes a first-order approximation to the risk-sharing condition for all i, similar to (7), resulting in $\mathbb{E}_t \Delta c_{it+1} = r_t^* + \psi_{it}$. Integrating across $i \in [0,1]$ yields the solution for the world interest rate, $r_t^* = \mathbb{E}_t \Delta y_{Tt+1} - \bar{\psi}_t$, where $\mathbb{E}_t \Delta y_{Tt+1} = \hat{r}_t^*$. Finally, using the Euler equation for the domestic currency i bond with nominal return R_{it}, we show that ψ_{it} defined in the lemma equals the currency i UIP deviation, $\psi_{it} = \log(R_{it}/R_t^*) - \mathbb{E}_t \Delta e_{it+1}$.

where $\hat{q}_{it} \equiv a_{it} - \hat{c}_{it}$, and the country budget constraints $\beta b^*_{it} - b^*_{it-1} = -\hat{z}_{it}$ for all $i \in [0,1]$. This extends the non-cooperative policy problem of a small open economy (15), with (24) generalizing (12). Recall that the local country i policymaker, if unconstrained, would optimally eliminate the UIP deviation, $\psi_{it} = 0$, taking r^*_t as given (Proposition 1). However, such policy may not be optimal from the perspective of a global policymaker who takes into account the endogeneity of r^*_t.

We prove in the appendix that the *cooperatively optimal* unconstrained FX interventions in country i ensure $\mathbb{E}_t \Delta \hat{z}_{it+1} = \psi_{it} - \bar{\psi}_t = 0$, which has the following immediate implications:

Proposition 7 (a) *If all countries are unconstrained, non-cooperative optimal FXI implement the global planner's allocation, and in particular $\psi_{it} = \bar{\psi}_t = 0$ for all $i \in (0,1]$, eliminating all UIP deviations and implementing $r^*_t = \hat{r}^*_t$.* (b) *When FXI are constrained in a subset of countries, non-cooperative policy is subject to an externality. The cooperative policy does not fully eliminate UIP deviations in the unconstrained countries, $\psi_{it} = \bar{\psi}_t \neq 0$, limiting inefficient capital outflows from/to constrained economies.*

When all countries are unconstrained, the optimal non-cooperative policies from Proposition 1 that eliminate UIP deviations country-by-country, $\psi_{it} = 0$ for all i, translate into a globally optimal outcome with $\bar{\psi}_t = 0$ and $r^*_t = \hat{r}^*_t$. That is, the Nash equilibrium played by local policymakers results in zero output gap and optimal risk sharing between all economies. Elimination of all UIP deviations rebalances capital flows and eliminates the pressure on the global real interest rate.[46] This result suggests the usefulness of *swap lines* between central banks, which can relax constraints on FXI and allow countries to achieve the optimum allocation without relying on either ex ante, or ex post international wealth transfers (Bahaj and Reis 2021). Furthermore, such swap lines are not subject to incentive compatibility or time consistency issues, as the best non-cooperative use of relaxed FXI does not lead to negative international spillovers and is beneficial cooperatively.

In contrast, when FXI of a subset of countries are constrained and shocks have a correlated component, so that $\bar{\psi}_t \neq 0$, this results in international spillovers that are not internalized by national policymakers. The cooperative policy eliminates the risk-sharing wedge between the group of unconstrained and constrained countries, reducing the extent of inefficient capital flows. For example, with a global demand shock for dollars $\bar{\psi}_t > 0$, the cooperative policy ensures $\mathbb{E}_t \Delta \hat{z}_{it+1} = 0$ for the unconstrained countries, yet with a UIP deviation against the dollar, $\psi_{it} = \bar{\psi}_t > 0$.[47] For comparison, a country's non-cooperative policy that eliminates the UIP deviation, $\psi_{it} = 0$, results in $\mathbb{E}_t \Delta \hat{z}_{it+1} = -\bar{\psi}_t < 0$.

Therefore, the cooperative policy under-reacts to the UIP wedge in order to curb inefficient capital

[46] Indeed, there are no first-order externalities in our environment when countries choose consumption of tradables subject to intertemporal budget constraint. Although international asset markets are incomplete, the fact that there is only one tradable good implies that there is no pecuniary externality (Geanakoplos and Polemarchakis 1986). Similarly, there is no aggregate demand externality for risk sharing when monetary policy closes the output gap (*cf.* Farhi and Werning 2016). This result contrasts with the inefficient non-cooperative equilibrium in Fanelli and Straub (2021) when countries participate in a "rat race" of reserve accumulation in a second-best world with redistributive FX interventions.

[47] For concreteness, consider a measure $m_0 > 0$ of the world economy, corresponding to the US and dollar pegs combined, with $\psi_{it} = 0$ for $i \in [0, m_0]$. A measure $m_1 > 0$ of countries, $i \in (m_0, m_0 + m_1]$, are constrained and face a correlated dollar demand shock against their currencies, $\bar{\psi}^c_t \equiv \frac{1}{m_1} \int_{m_0}^{m_0+m_1} \psi_{it} \mathrm{d}i > 0$. The remaining countries, $i \in (m_0 + m_1, 1]$ are unconstrained, and adopt the cooperative policy $\psi_{it} = \bar{\psi}_t = \frac{m_1}{m_0+m_1} \bar{\psi}^c_t$, where the last equality is the equilibrium fixed point. Thus, unconstrained economies mimic the average behavior of the other countries $-$ $\psi_{it} = 0$ of the dollarized economy with measure m_0 and $\bar{\psi}^c_t$ of the constrained countries with measure m_1.

inflows from the constrained economies, emphasizing the *complementarity* in the use of FXI across countries. Specifically, unconstrained FXI f^*_{it} respond to both domestic currency demand shocks n^*_{it} and currency demand shocks in constrained economies n^*_{jt} such that $n^*_{it}+f^*_{it}$ and $n^*_{jt}+f^*_{jt}$ — and, thus, ψ_{it} and ψ_{jt} — comove. Interestingly, this amplifies the equilibrium effect of the shock on $\bar{\psi}_t$ and r^*_t, resulting in larger capital inflows and current account deficits in the US, yet mitigates the aggregate outflow from the constrained economies.

Dominant currency spillovers We close this section by briefly considering spillovers from US monetary policy in a non-cooperative equilibrium characterized by Proposition 4. The multi-country setup clarifies the central role of the bilateral exchange rates against the dollar when implementing a crawling peg and explains why most countries in the world — including the ones with weak trade linkages to the US — use the dollar as an anchor currency in their monetary and FX policies (Ilzetzki, Reinhart, and Rogoff 2019).[48] Indeed, pegging to other currencies or baskets of currencies is suboptimal and can potentially exacerbate the risk-sharing wedge by increasing $\bar{\sigma}^2_{it}$. The asymmetric role of the US dollar exchange rate is not due to the specific form of currency market segmentation, but rather the assumption that the dollar is the international funding currency.

The immediate implication of the dollar dominance is the highly asymmetric spillovers of the US monetary policy. For example, a tightening of the US monetary stance lowers the dollar price of tradables p^*_{Tt} and leads to an appreciation of the dollar, $e_{it}\uparrow$ for $i \in (m_0, 1]$. To stabilize the exchange rate against the dollar and reduce the risk-sharing wedge, other countries are required to lean against the wind and raise their interest rates, which leads to a negative output gap $x_{it} < 0$. Thus, even when the US economy is small ($m_0 = 0$), all countries import its monetary stance giving rise to the global monetary cycle (Rey 2013, Egorov and Mukhin 2023).[49]

5 Robustness

The baseline model makes several strong assumptions to get a sharp characterization of the optimal policy. This section relaxes some of these assumptions — in particular, allowing for staggered price adjustment, expenditure switching in tradable goods, and incomplete pass-through — to evaluate robustness of our baseline results. Detailed derivations are relegated to Appendix A5.

5.1 Staggered prices

The assumption of fully rigid prices in the baseline analysis emphasizes our focus on the tradeoff between the output gap and international risk sharing. As a result, it removes domestic inflation as a

[48] Similarly to Hassan, Mertens, and Zhang (2023), the goal of the peg in our model is to eliminate the UIP deviation, but the anchor status of the dollar is due to the structure of financial markets, not the size of the US economy.

[49] This contrasts with a more symmetric global financial system under a gold standard. If gold is used as the international funding vehicle, the risk-sharing conditions remain the same except that the relevant measure of risk $\bar{\sigma}^2_{it}$ is now the volatility of exchange rates against gold. Given a zero nominal return on gold, $R^*_t = 1$, the price of tradables p^*_{Tt} (in units of gold) adjusts to implement the real rate of return r^*_t required to clear the goods market. This eliminates the asymmetry of a dominant currency, yet may lead to larger risk-sharing wedges due to a more volatile unit of account.

policy consideration. We now generalize our results to an environment with staggered price adjustment. In particular, we assume that there is a continuum of varieties of non-tradable goods with an elasticity of substitution $\varepsilon > 1$ that are produced by monopolistic competitors. Firms are subject to a Calvo (1983) friction and update prices with probability $1 - \lambda$ each period. We allow for markup shocks ν_t and assume that a constant production subsidy is used to eliminate the steady-state markup wedge.

The resulting planner's problem is largely isomorphic to the baseline problem (15), but features both non-tradable price inflation $\pi_{Nt} \equiv \Delta \log P_{Nt}$ and the output gap x_t in the welfare loss function:

$$\frac{1}{2}\,\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[\gamma z_t^2 + (1-\gamma)(x_t^2 + \alpha \pi_{Nt}^2)\big], \tag{25}$$

as well as an additional New-Keynesian Phillips curve (NKPC) constraint on the planner:

$$\pi_{Nt} = \kappa x_t + \beta \mathbb{E}_t \pi_{Nt+1} + \nu_t, \tag{26}$$

where $\alpha \equiv \varepsilon/\lambda$ is the relative weight on inflation in the welfare loss and $\kappa = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$ is the slope of the NKPC. The planner is still subject to the country budget constraint (11) and the risk-sharing condition (13). In contrast, the expenditure switching condition (2) now allows for real exchange rate adjustment by means of domestic non-tradable inflation, and hence we replace (12) with:

$$\Delta e_t = \Delta \tilde{q}_t + \Delta x_t - \Delta z_t + \pi_{Nt}. \tag{27}$$

Note that this constraint is equivalent to writing $\bar{\sigma}_t^2 = \mathrm{var}_t(\tilde{q}_{t+1} + x_{t+1} - \Delta z_{t+1} + \pi_{Nt+1})$ in the risk-sharing condition (13).

To summarize, with staggered price adjustment, the planner minimizes (25) subject to (11), (13), (26) and (27). By examining this problem, we see that the first best result in Propositions 1 generalizes to this case, with optimal FXI $f_t^* = -n_t^*$ still eliminating the UIP deviation and the risk-sharing wedge, $z_t = 0$, and optimal monetary policy choosing the optimal path of inflation and the output gap $\{x_t, \pi_{Nt}\}$, as in the closed economy. In the absence of markup shocks in the NKPC, $\nu_t = 0$, optimal monetary policy delivers $x_t = \pi_{Nt} = 0$. If in addition to $\nu_t = 0$ we also have $\Delta \tilde{q}_t = 0$ (or, equivalently, $\tilde{q}_t = \bar{q}$) then the divine coincidence result of Proposition 2 holds as before. Specifically, by fixing the nominal exchange rate, $e_t = \bar{q}$, the planner can simultaneously eliminate all gaps delivering the first-best allocation $z_t = x_t = \pi_{Nt} = 0$ with a single policy instrument — the monetary peg. Therefore, our open economy divine coincidence result extends the closed economy result by constructing the case where there is no conflict either domestically (between inflation and output gap stabilization) or externally (between domestic goals and expenditure switching).

Perhaps more surprisingly, the optimal policy away from the first-best — in particular, the crawling peg result in Propositions 4 — also generalizes to the staggered price environment. To see this, notice that the only interaction between the domestic economy and international risk sharing comes via the nominal exchange rate depreciation in (27), which result in the $x_{t+1} + \pi_{Nt+1}$ term in the definition of $\bar{\sigma}_t^2$ in the constraint set. This implies that the planner's problem can be broken into two sequential problems. First, solve for the optimal path of $\{x_t, \pi_{Nt}\}$ given the path of aggregate demand $m_t \equiv x_t + \pi_{Nt}$, and, second, solve for the optimal trade-off between risk sharing z_t and domestic conditions summarized by m_t. The latter problem is the same as in the baseline model, except that the output losses x_t^2 are replaced with the overall welfare losses due to the output gap and inflation. This implies that the results about the second-best policies, including the optimal partial peg (16), extend to the setup with partial price adjustment.

5.2 Terms of trade and incomplete pass-through

Other important limitation of the baseline model are the assumptions of no law of one price (LOP) deviations for tradables, of the efficient terms of trade, and of the homogenous home and foreign tradable goods. Following the normative open-economy literature (Galí and Monacelli 2005, Devereux and Engel 2003, Benigno and Benigno 2003), in this extension we switch from a model with a non-tradable and a homogenous tradable good to a model with two tradable goods — a home good that is both consumed domestically C_{Ht} and exported abroad C_{Ht}^* and an imported foreign good C_{Ft}.

We maintain the assumption of log-linear preferences with $C_t = C_{Ht}^{1-\gamma} C_{Ft}^{\gamma}$, with P_{Ht} and P_{Ft} denoting the home-currency prices of the two goods. We further assume linear technology, $A_t L_t = C_{Ht} + C_{Ht}^*$, and CES demand for exports, $C_{Ht}^* = \gamma P_{Ht}^{*-\varepsilon} C_t^*$, where P_{Ht}^* is the export price in foreign currency, $\varepsilon > 1$ is the elasticity of foreign demand, and C_t^* is the foreign demand shifter. For simplicity, we assume that all prices are fully sticky in the currency of invoicing. Consistent with the evidence on international prices (Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller 2020), we assume that domestic prices are set in the local currency, while export prices are invoiced in dollars (DCP). Appendix A5 provides detailed derivations and also discusses the alternative case of producer currency pricing (PCP).

When trade prices are sticky in foreign currency, we have after normalization that $P_{Ht} = 1$, $P_{Ft} = \mathcal{E}_t$ and $P_{Ht}^* = 1$. That is, both export prices $\mathcal{E}_t P_{Ht}^*$ and import prices P_{Ft} — as well as the law of one price deviations $\mathcal{E}_t P_{Ht}^* / P_{Ht}$ — comove one-to-one with the nominal exchange rate $\mathcal{E}_t$. In contrast, the terms of trade are exogenous and stable independently of the shocks, $P_{Ft}/(\mathcal{E}_t P_{Ht}^*) = 1$, a trademark feature of DCP economies (Gopinath and Itskhoki 2022). We define the natural real exchange rate as $\tilde{Q}_t = \frac{\gamma}{1-\gamma} \frac{\tilde{C}_{Ht}}{\tilde{C}_{Ft}}$, where $(\tilde{C}_{Ht}, \tilde{C}_{Ft})$ is the first-best consumption allocation. As a result, the equilibrium condition in the goods market — the expenditure switching condition — is still given by (12), $e_t = \tilde{q}_t + x_t - z_t$, where now $x_t = c_{Ht} - \tilde{c}_{Ht}$ and $z_t = c_{Ft} - \tilde{c}_{Ft}$ are the domestic and import consumption wedges. Since under DCP the export quantity is demand-determined and exogenous to the policy, x_t still corresponds to the output gap (the production wedge) and z_t still corresponds to the risk-sharing wedge. The equilibrium condition in the financial market (13) also remains unchanged.

What changes, however, is the welfare loss function (10) and the country budget constraint (11).

Specifically, the planner's problem now becomes:

$$\min_{\{x_t,z_t,e_t,b_t^*,f_t^*,\bar{\sigma}_t^2\}} \frac{1}{2}\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)x_t^2+\gamma(\varepsilon-1)\tilde{q}_t^2\big],$$
$$\text{subject to (12), (13) and } \ \beta b_t^* = b_{t-1}^*-(\varepsilon-1)\tilde{q}_t - z_t.$$

Note the new term in the natural real exchange rate $\tilde{q}_t$ that represents both an additional source of welfare losses and a deviation from the first-best path of net exports due to DCP sticky trade prices. Movements in $\tilde{q}_t$ require an adjustment in the first-best terms of trade, which are constant under DCP price stickiness. As a result, the export wedge is exogenous to both monetary and FX policy (Egorov and Mukhin 2023), and the policymaker takes it as given and solves for the optimal path of endogenous wedges $\{x_t, z_t\}$.

While it is still optimal to use monetary policy to close the output gap, $x_t = 0$, it is no longer possible to ensure the first-best path of imports and NFA, $z_t = b_t^* = 0$, when $\tilde{q}_t$ fluctuates. This is because movements in $\tilde{q}_t$ result in deviations of export revenues from their first-best path, and thus imports must adjust. Nonetheless, it is still optimal to use FXI to offset currency demand shocks, $f_t^* = b_t^* - n_t^*$, and eliminate UIP deviations, ensuring $\mathbb{E}_t\Delta z_{t+1} = 0$. In this sense the results of Proposition 1 generalize to this case. Interestingly, the divine coincidence from the baseline model (Proposition 2) extends fully to the DCP economy: if the natural real exchange rate is stable, $\tilde{q}_t = 0$, monetary policy alone can implement the first-best allocation, $x_t = z_t = b_t^* = 0$. Indeed, in this case, constant terms of trade are efficient and there is no export wedge. Finally, notice that exogenous shocks $\tilde{q}_t$ in the country budget constraint do not affect any optimality conditions and, therefore, Theorems 1 and 2 still apply in this setting. In particular, the second-best monetary policy (Proposition 4) still partially pegs the exchange rate balancing the costs of the output gap x_t and risk-sharing z_t wedges.

Exchange rate disconnect Our baseline model assumes a unit elasticity of substitution and a complete exchange rate pass-through into tradable prices. As we argue in Itskhoki and Mukhin (2021a), both the low elasticity of substitution and the low pass-through elasticity facilitate the model's ability to match the disconnect in volatilities between exchange rates and macro aggregates in response to financial shocks n_t^*. In Appendix A5, we extend the baseline model to features CES demand between tradables and non-tradables with elasticity $\theta > 0$, as well as strategic complementarities in price setting which generate pricing-to-market and incomplete pass-through $\alpha \in (0, 1)$ for tradable goods. We show that the planner's problem remains the same as in (15) except for the goods market equilibrium condition (12), which now becomes:

$$e_t = \frac{1}{\alpha\theta}\big(\theta\tilde{q}_t + x_t - z_t\big), \qquad \text{where} \qquad \tilde{q}_t \equiv \frac{1}{\theta}(\tilde{c}_{Nt} - \tilde{c}_{Tt}).$$

Thus, both incomplete pass-through $\alpha < 1$ and low elasticity of substitution θ imply that large movements in the exchange rate are associated with muted expenditure switching effects in proportion with $\alpha\theta$ and, hence, lower volatility in macro quantities. At the same time, the policy problem remains isomorphic to the baseline problem (15), and all our optimal policy results continue to hold.

6 Conclusion

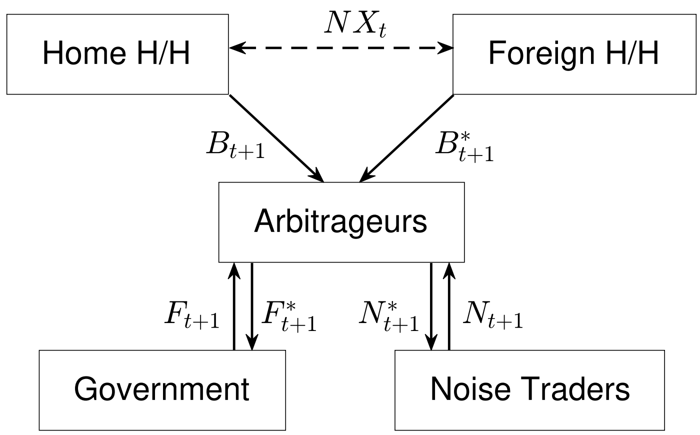
We study optimal exchange rate policy in an open economy with nominal rigidities in the goods market and intermediation frictions in the international financial market. In contrast to the previous normative literature, we use a framework that is consistent with major exchange rate properties (puzzles), including evidence on the shifts between floating and fixed exchange rate regimes. We develop a tractable policy analysis framework that admits an intuitive linear-quadratic approximation to the planner's problem, yet is rich enough to accommodate the key policy trade-offs, and in particular provides a rationale for the "fear of floating" observed in many small open economies.

We show that the first-best allocation can be implemented with inward-looking monetary policy targeting inflation to close the output gap in the goods market and FX interventions targeting frictional UIP deviations in the asset market. Whenever FX policies are constrained or the UIP target is unobservable, monetary policy is optimally used to balance out the goods market and financial market wedges, with the weight on the latter increasing in the openness of the economy. When the natural (first-best) real exchange rate is stable, both objectives can be achieved by means of a monetary peg, or an optimal currency area. In this knife-edge benchmark, the goods market objectives do not need to be compromised for the risk-sharing benefits associated with a stable exchange rate. More generally, optimal monetary policy implements a managed float or crawling peg/band by leaning against exchange rate surprises, and more intensively so in periods of large capital (out)flows associated with frictional UIP deviations. We, further, discuss the benefits of forward-guidance and preemptive FX policies, as well as complementarity of FX interventions in a cooperative global economy. Finally, we show that capital controls are not necessary unless the policymaker wants to maximize its monopoly rents in the home currency market.

Our analysis lays out promising avenues for future research emphasizing the main objects and parameters for empirical measurement. This includes developing techniques to evaluate the key targets of monetary and FX policy — the natural level of the real exchange rate and frictional UIP deviations (Bekaert 1995, Kollmann 2005, Engel 2016, Kalemli-Özcan and Varela 2021). More work is also needed to evaluate the elasticity of currency demand and how it depends on exchange rate regimes and varies across countries (Hau, Massa, and Peress 2009, Koijen and Yogo 2020, Bahaj and Reis 2023, Beltran and He 2023). We also hope that the approximation methods developed in this paper can be applied in other environments, and in particular in the analysis of the effects of monetary policy on risk premia and credit spreads of other assets beyond currency markets (Ray 2019, Caballero and Simsek 2022).

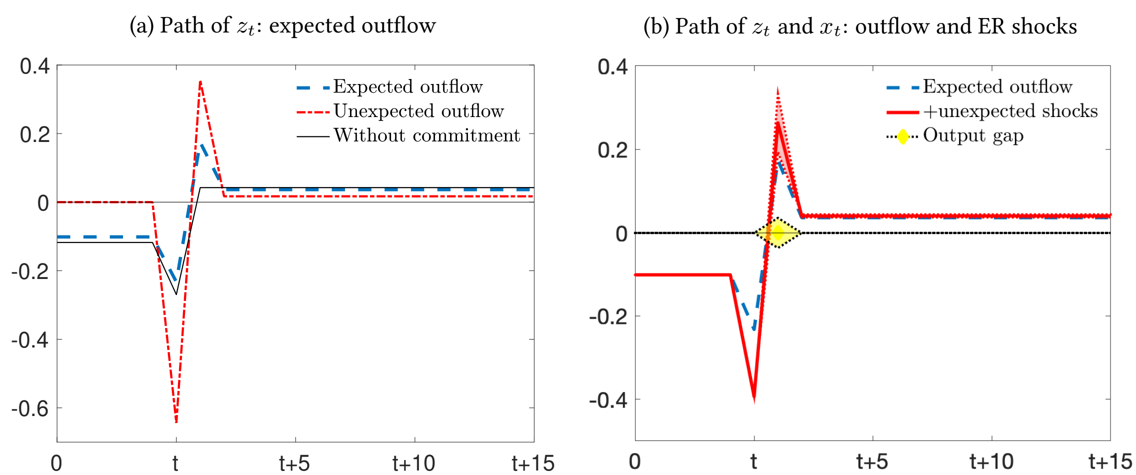
A Appendix

Figure A1: Structure of currency markets



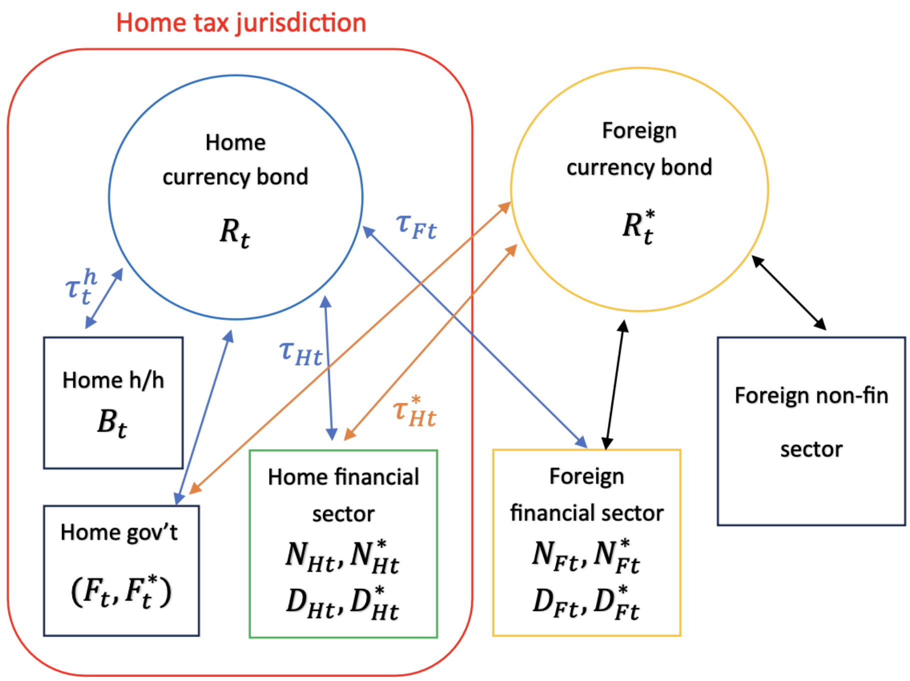
Note: the figure illustrates the structure of currency markets in the baseline version of the model. Home (foreign) households can only trade home (foreign) currency bonds B_t (B_t^*), while the other agents exchange bonds in one currency for bonds in the other currency: noise traders demand shocks N_t, N_t^* are exogenous, arbitrageurs choose their portfolios D_t, D_t^* to maximize mean-variance preferences, and the government uses sterilized FX interventions F_t, F_t^* to maximize social welfare.

Figure A2: Optimal capital flows and output gaps



Note: The figure plot the optimal allocation $\{x_t, z_t\}$ when FXI are unconstrained in all periods but t, i.e. $\mu_{t+j} = 0$ for all $j \neq 0$, and there is an expected unaccommodated capital outflow at t, $\mathbb{E}_t \Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*) > 0$. Left panel describes the case when capital outflow is expected, $\mathbb{E}_0 n_t^* = n_t^* > 0$, and there are no further shocks. Right panel considers the case where $n_t^* > \mathbb{E}_{t-j} n_t^* > 0$ and $e_{t+1} \neq \mathbb{E}_t e_{t+1}$, requiring the response of (x_{t+1}, z_{t+1}) to the exchange rate surprise. The full analytical solution is provided in the end of Appendix A3.

Figure A3: Currency markets with capital controls



Note: the figure illustrates the structure of currency markets in the extension with capital controls: households pay τ_t^h on their savings, domestic intermediaries pay respectively τ_{Ht} and τ_{Ht}^* on their home- and foreign-currency positions, foreign intermediaries pay τ_{Ft} on their home-currency positions (see Section 4.1 for details).

A1 Exact non-linear policy problem

As described in Sections 2.1–2.2, the Ramsey problem maximizes the household welfare in (1) over policies $\{R_t, F_t^*\}$ and the equilibrium allocation $\{C_{Nt}, C_{Tt}, B_t^*, \mathcal{E}_t\}$ and $\{\sigma_t^2\}$, subject to the equilibrium system (2)–(3) and (6)–(7) (including the definition of σ_t^2), given the stochastic path of exogenous variables $\{A_t, Y_{Tt}, R_t^*, N_t^*\}$ and subject to initial and transversality conditions on B_t^*:

$$\max_{\{R_t,F_t^*,C_{Nt},C_{Tt},B_t^*,\mathcal{E}_t,\sigma_t^2\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma)\left(\log C_{Nt} - \frac{C_{Nt}}{A_t}\right)\right] \tag{A1}$$

$$\begin{aligned}
\text{subject to} \qquad \frac{B_t^*}{R_t^*} - B_{t-1}^* &= Y_{Tt} - C_{Tt}, \\
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} &= 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \\
\beta R_t \mathbb{E}_t \frac{C_{Nt}}{C_{Nt+1}} &= 1, \\
\mathcal{E}_t &= \frac{\gamma}{1-\gamma}\frac{C_{Nt}}{C_{Tt}}, \\
\sigma_t^2 &= R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big),
\end{aligned}$$

where we used the non-tradable production function and market clearing $C_{Nt} = Y_t = A_t L_t$ to substitute for L_t in the welfare function.[50]

First best The first-best allocation maximizes (A1) with respect to $\{C_{Nt}, C_{Tt}, B_{t+1}^*\}$ and subject to the budget constraint only, removing the remaining four constraints. The optimality conditions for this problem imply $\tilde{C}_{Nt} = A_t$ and $\{\tilde{C}_{Tt}, \tilde{B}_{t+1}^*\}$ such that:

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1$$

and the budget constraint holds, $B_t^*/R_t^* - B_{t-1}^* = Y_{Tt} - C_{Tt}$.

When the two policy instruments — monetary policy and FXI, $\{R_t, F_t^*\}$ — are available and unconstrained, the first best allocation is feasible. This is because the two constraints on the policy problem (A1) — namely, the two Euler equations (with R_t and R_t^*, respectively), with the last two constraints being static side equations defining $\mathcal{E}_t$ and σ_t^2 — each feature an independent policy instrument which can ensure that the respective constraint is relaxed.

[50]Note that one can alternatively rewrite the problem in terms of wages W_t, as in an equilibrium with sticky prices $P_{Nt} = 1$ the labor supply condition implies $W_t = P_{Nt}C_{Nt} = C_{Nt}$, and monetary policy by controlling aggregate nomiunal expenditure P_tC_t, controls also the path of nominal wages, $W_t = P_{Nt}C_{Nt} = (1-\gamma)P_tC_t$.

Specifically, decentralizing the first-best allocation requires a path of $\{R_t, F_t^*\}$ such that:

$$\beta R_t \mathbb{E}_t \frac{A_t}{A_{t+1}} = 1,$$
$$\omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*} = 0$$

and the implies path of the nominal exchange rate given by $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma}\frac{A_t}{\tilde{C}_{Tt}}$. The two displayed equations characterize the necessary path of policy outcomes $\tilde{R}_t$ and $\tilde{F}_t^*$, leaving aside the conventional issue of uniqueness of the decentralized equilibrium (see Atkeson, Chari, and Kehoe 2010). Therefore, the first-best monetary policy eliminates the output gap, that is, ensures $C_{Nt} = \tilde{C}_{Nt} = A_t$, while the first-best financial market policy ensures a zero risk-sharing wedge. This happens when either $\omega\sigma_t^2 = 0$, or when $F_t^* = B_t^* - N_t^*$; the latter corresponds to the case of Proposition 1, while the former to divine coincidence of Proposition 2 (or trilemma models with $\omega = 0$).[51] The first-best path of NFA according to the budget constraint is $\tilde{B}_t^* = R_t^*(\tilde{B}_{t-1}^* + Y_{Tt} - \tilde{C}_{Tt})$, and hence the optimal FXI is $\tilde{F}_t^* = \tilde{B}_t^* - N_t^*$ when $\sigma_t^2 = \tilde{\sigma}_t^2 = \tilde{R}_t^2 \cdot \mathrm{var}_t\big(\tilde{\mathcal{E}}_t/\tilde{\mathcal{E}}_{t+1}\big) \neq 0$.

Optimality conditions We make the following substitution of variables:

$$\Gamma_t \equiv \frac{1}{C_{Nt}}, \qquad \beta R_t \mathbb{E}_t \frac{\Gamma_{t+1}}{\Gamma_t} = 1, \qquad \mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{1}{\Gamma_t C_{Tt}}, \tag{A2}$$

where the last two conditions are implied by the constraints in (A1). We can thus recover the path of $\{R_t, C_{Nt}, \mathcal{E}_t\}$ from the path of $\{\Gamma_t, C_{Tt}\}$. As a result, the original policy problem (A1) is equivalent to:

$$\max_{\{\Gamma_t, C_{Tt}, B_t^*, \sigma_t^2\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} - (1-\gamma)\left(\log \Gamma_t + \frac{1}{A_t \Gamma_t}\right)\right] \tag{A1$'$}$$
$$\text{subject to} \qquad \frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$$
$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*},$$
$$\sigma_t^2 \beta^2 C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2 = \mathbb{E}_t(\Gamma_{t+1} C_{Tt+1})^2 - (\mathbb{E}_t \Gamma_{t+1} C_{Tt+1})^2,$$

where we used (A2) to solve out $\{R_t, \mathcal{E}_t\}$ from the definition of σ_t^2:

$$\sigma_t^2 = \overbrace{\big(\beta \mathbb{E}_t[\Gamma_{t+1}/\Gamma_t]\big)^{-2}}^{=R_t^2} \cdot \overbrace{(\Gamma_t C_{Tt})^{-2}\big[\underbrace{\mathbb{E}_t(\Gamma_{t+1}C_{Tt+1})^2 - (\mathbb{E}_t \Gamma_{t+1} C_{Tt+1})^2}_{=\mathrm{var}_t(\Gamma_{t+1}C_{Tt+1})}\big]}^{=\mathrm{var}_t(\mathcal{E}_t/\mathcal{E}_{t+1})},$$

resulting in the final constraint in (A1$'$). Note that we characterize the planner's optimality conditions for an arbitrary path of $\{F_t^*\}$, and then discuss the optimal path of $\{F_t^*\}$.

[51] Divine coincidence, of course, requires that $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma}\frac{A_t}{\tilde{C}_{Tt}} = const$; otherwise, at least one of the two wedges cannot be eliminated — either $\sigma_t^2 \neq 0$ and hence $C_{Tt} \neq \tilde{C}_{Tt}$ (for an arbitrary path of $F_t^* \neq \tilde{F}_t^*$), or $C_{Nt} \neq \tilde{C}_{Nt} = A_t$ under the peg (with $\sigma_t^2 = 0$ that ensures $C_{Tt} = \tilde{C}_{Tt}$ for any path of F_t^*).

We write the Lagrangian for (A1′) as follows:

$$\mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\gamma \log C_{Tt} - (1-\gamma)\Big(\log \Gamma_t + \frac{1}{A_t \Gamma_t} \Big) \\ + \Lambda_t \Big(B^*_{t-1} + Y_{Tt} - C_{Tt} - \frac{B^*_t}{R^*_t} \Big) \\ + M_t \Big(1 + \omega \sigma_t^2 \frac{B^*_t - N^*_t - F^*_t}{R^*_t} - \beta R^*_t \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} \Big) \\ + D_t \Big(\beta^2 \sigma_t^2 C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2 - \mathbb{E}_t (\Gamma_{t+1} C_{Tt+1})^2 + (\mathbb{E}_t \Gamma_{t+1} C_{Tt+1})^2 \Big) \Big],$$

where $\{\Lambda_t, M_t, D_t\}$ is the sequence of Lagrange multipliers on the respective constraints. The first order conditions with respect to $\{\Gamma_t, C_{Tt}, B^*_t, \sigma_t^2\}$ are as follows:[52]

$$0 = -(1-\gamma)\frac{1}{\Gamma_t} + (1-\gamma)\frac{1}{A_t \Gamma_t^2} + 2\beta^{-1} D_{t-1} C_{Tt} \Big(\beta^2 \sigma_{t-1}^2 \frac{C_{Tt-1}^2}{C_{Tt}} \mathbb{E}_{t-1}\Gamma_t - \Gamma_t C_{Tt} + \mathbb{E}_{t-1}(\Gamma_t C_{Tt}) \Big),$$
$$0 = \frac{\gamma}{C_{Tt}} - \Lambda_t - M_t \beta R^*_t \mathbb{E}_t \frac{1}{C_{Tt+1}} + M_{t-1} R^*_{t-1} \frac{C_{Tt-1}}{C_{Tt}^2} + 2 D_t \beta^2 \sigma_t^2 C_{Tt} (\mathbb{E}_t \Gamma_{t+1})^2 - 2\beta^{-1} D_{t-1} \Gamma_t (\Gamma_t C_{Tt} - \mathbb{E}_{t-1}(\Gamma_t C_{Tt})),$$
$$0 = -\frac{\Lambda_t}{R^*_t} + \beta \mathbb{E}_t \Lambda_{t+1} + M_t \frac{\omega \sigma_t^2}{R^*_t},$$
$$0 = M_t \omega \frac{B^*_t - N^*_t - F^*_t}{R^*_t} + D_t \beta^2 C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2.$$

where we define $D_{-1} = M_{-1} = 0$, and we use the fact that operator $\mathbb{E}_t\{\cdot\}$ sums across future realizations of uncertainty using conditional probabilities $\pi(h^{t+1})/\pi(h^t)$ for any history of exogenous states $h^{t+1} \equiv \{A_s, Y_{Ts}, R^*_s\}_{s=0}^{t+1}$

We simplify the conditions as follows. The last two conditions allow to relate the Lagrange multipliers Λ_t and D_t with M_t:

$$\Lambda_t - \beta R^*_t \mathbb{E}_t \Lambda_{t+1} = M_t \omega \sigma_t^2, \tag{A3}$$

$$D'_t = M_t \omega (\mathcal{E}_t R_t)^2 \frac{N^*_t + F^*_t - B^*_t}{R^*_t}, \tag{A4}$$

where we used definitions (A2) in the second line and substituted $D'_t \equiv \big(\frac{\gamma}{1-\gamma}\big)^2 D_t$. Next we simplify the first optimality condition by substituting out σ_{t-1}^2 using its definition (the third constraint of the problem):

$$\beta(1-\gamma)(X_t - 1) = 2D'_{t-1} \left[\frac{1}{\mathcal{E}_t^2} - \frac{\mathbb{E}_{t-1}\mathcal{E}_t^{-1}}{\mathcal{E}_t} - \frac{\Gamma_t}{\mathbb{E}_{t-1}\Gamma_t} \Big(\mathbb{E}_{t-1}\mathcal{E}_t^{-2} - (\mathbb{E}_{t-1}\mathcal{E}_t^{-1})^2 \Big) \right], \tag{A5}$$

where we defined $X_t \equiv C_{Nt}/A_t = 1/(A_t \Gamma_t)$ so that $X_t - 1$ corresponds to the output gap. Note that (A5) implies $\mathbb{E}_{t-1} X_t = 1$, as the conditional expectations of the right-hand side is zero. The final optimality condition:

[52] Note that with an optimal unconstrained choice of F^*_t at t, we additionally have that $M_t = 0$, and therefore $D_t = 0$, $C_{Nt+1} = 1/\Gamma_{t+1} = A_{t+1}$, $\Lambda_t = \beta R^*_t \mathbb{E}_t \Lambda_{t+1}$ and $\beta R^*_t \mathbb{E}_t [C_{Tt}/C_{Tt+1}] = 1$, consistent with conditions for the first best allocation.

$$\gamma\Big(1-\frac{\Lambda_t}{\gamma/C_{Tt}}\Big)-(1-\gamma)(X_t-1)=M_t\beta R_t^*\mathbb{E}_t\frac{C_{Tt}}{C_{Tt+1}}-M_{t-1}R_{t-1}^*\frac{C_{Tt-1}}{C_{Tt}}-\frac{2D_t'\sigma_t^2}{(\mathcal{E}_tR_t)^2}+\frac{2D_{t-1}'\sigma_{t-1}^2}{\mathcal{E}_{t-1}^2R_{t-1}}\frac{\Gamma_t}{\Gamma_{t-1}} \tag{A6}$$

$$=M_t\Big(\beta R_t^*\mathbb{E}_t\frac{C_{Tt}}{C_{Tt+1}}+2\omega\sigma_t^2\frac{D_t^*}{R_t^*}\Big)-\beta^{-1}M_{t-1}\Big(\beta R_{t-1}^*\frac{C_{Tt-1}}{C_{Tt}}+2\omega\sigma_{t-1}^2\frac{D_{t-1}^*}{R_{t-1}^*}\frac{\Gamma_t}{\mathbb{E}_{t-1}\Gamma_t}\Big),$$

where we used $D_t^*=B_t^*-N_t^*-F_t^*$.

Conditions (A3)–(A6) together with definitions in (A2) and constraints in (A1′) characterize the optimal monetary policy for a given path of FXI $\{F_t^*\}$ and the associated allocation.

A2 Approximations

A2.1 Second-order approximation to the objective function

Consider any allocation $\{C_{Nt},C_{Tt},L_t,B_t^*\}$ that satisfies production possibilities frontier for non-tradables, $C_{Nt}=A_tL_t$, and the country budget constraint (6):

$$\frac{B_t^*}{R_t^*}=B_{t-1}^*+Y_{Tt}-C_{Tt},$$

for a givem B_{-1}^* and a transversality condition on B_∞^*, and corresponding to a stochastic path of shocks $\{A_t,Y_{Tt},R_t^*\}$. We refer to all such allocation as resource- and budget-feasible. The first best allocation corresponding to the same path of shocks is denoted with $\{\tilde C_{Nt},\tilde C_{Tt},\tilde L_t,\tilde B_t^*\}$, it is also resource- and budget-feasible, and satisfies the following optimality conditions (see Appendix A1):

$$\tilde C_{Nt}=A_t,\qquad \tilde L_t=1,\qquad \beta R_t^*\mathbb{E}_t\frac{\tilde C_{Tt}}{\tilde C_{Tt+1}}=1.$$

A non-stochastic zero-NFA steady state corresponding to $(\bar A,\bar Y_T,\bar R^*)$ such that $\bar R^*=1/\beta$, is given by $(\bar C_N,\bar C_T,\bar L,\bar B^*)$:

$$\bar C_N=\bar A,\qquad \bar L=1,\qquad \bar C_T=\bar Y_T,\qquad \bar B^*=0,$$

which implies $\overline{NX}=\bar Y_T-\bar C_T=0$ and the steady state budget constraint is satisfied. Finally, the welfare function is given by (1).

Lemma A1 *The second order Taylor expansion around a zero-NFA steady state $(\bar C_N,\bar C_T,\bar L)$ of the welfare loss for any budget- and resource-feasible allocation $\{C_{Nt},C_{Tt},L_t\}$ relative to the first-best allocation $\{\tilde C_{Nt},\tilde C_{Tt},\tilde L_t\}$ is given by:*

$$\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)x_t^2\big],$$

where $z_t=\log(C_{Tt}/\tilde C_{Tt})$ and $x_t=\log(C_{Nt}/\tilde C_{Nt})$. Therefore, it is sufficient to know the first-order dynamics of the two wedges $\{x_t,z_t\}$ to evaluate the second-order welfare loss.

Proof: We take a second order Taylor expansions of (1) for any resource- and budget-feasible allocation $\{C_{Nt},C_{Tt},L_t,B_t;A_t,Y_{Tt},R_t^*\}$ around a zero-NFA steady state $(\bar C_N,\bar C_T,\bar L,\bar B^*)=(\bar A,\bar Y_T,1,0)$, using log deviations:

$$c_{Nt} = \log(C_{Nt}/\bar{C}_N), \quad c_{Tt} = \log(C_{Tt}/\bar{C}_T), \quad a_t = \log(A_t/\bar{A}), \quad y_{Tt} = \log(Y_{Tt}/\bar{Y}_T), \quad r_t^* = \log(R_t^*/\bar{R}^*),$$

and for NFA we use a proportional deviation relative to $\bar{Y}_T$:

$$\check{b}_t^* = B_t^*/\bar{Y}_T.$$

Note that the first best deviation and the wedge for non-tradables are $\tilde{c}_{Nt} = \log(\tilde{C}_{Nt}/\bar{A}) = \log(A_t/\bar{A}) = a_t$ and $x_t = \log(C_{Nt}/\tilde{C}_{Nt}) = c_{Nt} - \tilde{c}_{Nt} = c_{Nt} - a_t$. We also use the fact that for any resource-feasible allocation $L_t = C_{Nt}/A_t$, and hence we solve out L_t from the welfare function.

We, therefore, can rewrite the welfare function (1) in terms of deviations as:

$$\mathbb{W}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\gamma \log \bar{Y}_T + (1-\gamma)(\log \bar{A} - 1) + \gamma c_{Tt} + (1-\gamma)\big[\log c_{Nt} - \big(e^{c_{Nt}-a_t} - 1\big)\big]\Big],$$

as well as the flow budget constraint (6) as:

$$\check{b}_{t-1}^* + e^{y_{Tt}} - e^{c_{Tt}} - \beta e^{-r_t^*} \check{b}_t^* = 0,$$

using the fact that $\bar{C}_T = \bar{Y}_T$ and $\bar{R}^* = 1/\beta$. We characterize the welfare loss in two steps:

1. The second-order Taylor expansion for the non-tradable terms in $\mathbb{W}_0$ is:

$$\begin{aligned}
\mathbb{E}_0 \Big[c_{Nt} - \big(e^{c_{Nt}-a_t} - 1\big)\Big] &= \mathbb{E}_0 \Big[\underbrace{c_{Nt} - (c_{Nt} - a_t)}_{=a_t} - \frac{1}{2}(\underbrace{c_{Nt} - a_t}_{=x_t})^2 \Big] + h.o.t. \\
&= \mathbb{E}_0 a_t - \frac{1}{2}\mathbb{E}_0 x_t^2 + h.o.t.,
\end{aligned}$$

   and in the first best allocation $x_t = 0$ as $\tilde{c}_{Nt} = a_t$.

2. The second-order Taylor expansion to the flow budget constraint is:

$$0 = \check{b}_{t-1}^* + y_{Tt} + \frac{1}{2}y_{Tt}^2 - c_{Tt} - \frac{1}{2}c_{Tt}^2 - \beta\check{b}_t^* + \beta r_t^* \check{b}_t^* + h.o.t.,$$

   which we use to express:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_{Tt} = \check{b}_{-1}^* + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[y_{Tt} + \frac{1}{2}y_{Tt}^2 - \frac{1}{2}c_{Tt}^2 + \beta r_t^* \check{b}_t^* \big] + h.o.t.,$$

   using the transversality condition for NFA deviations, $\lim_{j\to\infty} \beta^j \check{b}_{t+j} = 0$. Evaluating relative to the first-best allocation, we have:

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma\tilde{c}_{Tt}-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma c_{Tt}=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma\Big[\frac{1}{2}c_{Tt}^2-\frac{1}{2}\tilde{c}_{Tt}^2-\beta r_t^*b_t^*\Big]+h.o.t.$$

$$=\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma z_t^2+\underbrace{\mathbb{E}_0\textstyle\sum_{t=0}^{\infty}\beta^t\gamma\big[(c_{Tt}-\tilde{c}_{Tt})\tilde{c}_{Tt}-\beta r_t^*b_t^*\big]}_{=0+h.o.t.}+h.o.t.,$$

where we expanded $c_{Tt}=z_t+\tilde{c}_{Tt}$ and denoted with $b_t^*=\check{b}_t^*-\tilde{b}_t^*=(B_t^*-\tilde{B}_t^*)/\bar{Y}_T$ the proportional deviation of the NFA position from the first best NFA. Finally, we show that:

$$0=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma\big[(c_{Tt}-\tilde{c}_{Tt})\tilde{c}_{Tt}-\beta r_t^*b_t^*\big]+h.o.t.$$

$$=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[(b_{t-1}^*-\beta b_t^*)\tilde{c}_{Tt}-\beta r_t^*b_t\big]+h.o.t.$$

$$=\underbrace{b_{-1}^*}_{=0}\cdot\tilde{c}_{T0}+\beta\sum_{t=0}^{\infty}\beta^t\underbrace{\mathbb{E}_0\big[b_t^*(\Delta\tilde{c}_{Tt+1}-r_t^*)\big]}_{=0+h.o.t.}+h.o.t.$$

The second line uses the expansion of the flow budget constraint for c_{Tt} and $\tilde{c}_{Tt}$, which implies:

$$(c_{Tt}-\tilde{c}_{Tt})\tilde{c}_{Tt}=\Big(b_{t-1}^*+\frac{1}{2}\tilde{c}_{Tt}^2-\frac{1}{2}c_{Tt}^2-\beta b_t^*+\beta r_t^*b_t^*+h.o.t.\Big)\tilde{c}_{Tt}=(b_{t-1}^*-\beta b_t^*)\tilde{c}_{Tt}+h.o.t.$$

The third lines uses the fact that $b_{-1}^*=\check{b}_{-1}^*-\tilde{b}_{-1}^*=0$ by the initial condition, and the optimality condition (Euler equation) for the first-best consumption growth, which we rewrite in log deviations as $e^{r_t^*}\mathbb{E}_te^{-\Delta\tilde{c}_{Tt+1}}=1$, and take the following second-order Taylor expansion:

$$\mathbb{E}_t\Delta\tilde{c}_{Tt+1}-r_t^*=\frac{1}{2}(r_t^*)^2+\frac{1}{2}\mathbb{E}_t(\Delta\tilde{c}_{Tt+1})^2-r_t^*\mathbb{E}_t\Delta\tilde{c}_{Tt+1}+h.o.t.$$

and therefore using the law of iterated expectations:

$$\mathbb{E}_0\big[b_t^*(\Delta\tilde{c}_{Tt+1}-r_t^*)\big]=\mathbb{E}_0\big[b_t^*(\mathbb{E}_t\Delta\tilde{c}_{Tt+1}-r_t^*)\big]=0+h.o.t.$$

Combining these results, we evaluate the welfare loss relative to the first-best allocation to be given by $\tilde{\mathbb{W}}_0-\mathbb{W}_0=\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)x_t^2\big]$. ■

Alternative approach We next provide an alternative proof of Lemma A1 based on a new method of deriving quadratic loss functions. The central idea is to leverage the fact that the approximation is taken in deviations from the efficient allocation. The main advantage of the method is that it can be applied much more easily and requires fewer derivations. To economize space, we use this approach below to derive the loss function in most extensions of the baseline model.

To this end, consider a general optimization problem

$$\max_x \quad F(x,\varepsilon) \qquad \text{subject to} \quad G(x,\varepsilon)=0,$$

where x is a vector of endogenous variables, ε is a vector of shocks and $G(\cdot)$ is a vector function. The corresponding Lagrangian is defined by

$$\mathcal{L}(x, \lambda, \varepsilon) = F(x, \varepsilon) + \lambda G(x, \varepsilon).$$

For a given value of ε, let $\tilde{x} = \tilde{x}(\varepsilon)$ and $\tilde{\lambda} = \tilde{\lambda}(\varepsilon)$ denote its saddle point (solution). As a point of approximation consider $(x, \lambda, \varepsilon) = (\bar{x}, \bar{\lambda}, 0) \equiv \big(\tilde{x}(0), \tilde{\lambda}(0), 0\big)$ and use $\nabla F(x, \varepsilon) = \big\{\frac{\partial F}{\partial x^i}\big\}_i$ and $\nabla^2 F(x, \varepsilon) = \Big\{\frac{\partial^2 F}{\partial x^i \partial x^j}\Big\}_{i,j}$ to denote respectively the vector of first derivatives and the Hessian (with $\nabla G(x, \varepsilon)$ and $\nabla^2 G(x, \varepsilon)$ defined symmetrically).

Lemma A2 *For any feasible x, the second-order approximation of the loss function around $(\bar{x}, \bar{\lambda}, 0)$ is*

$$-\frac{1}{2} dx' \Big[\nabla^2 F(\bar{x}, 0) + \bar{\lambda} \nabla^2 G(\bar{x}, 0)\Big] dx, \quad \textit{where} \;\; dx \equiv x - \tilde{x}.$$

Proof: For any feasible allocation x, we have $G(x, \varepsilon) = 0$ and hence, $\mathcal{L}(x, \lambda, \varepsilon) = F(x, \varepsilon)$. Therefore, it is sufficient to focus on the quadratic approximation to the Lagrangian. With some abuse of notation, for a given value of ε, take the second-order Taylor expansion around $(\tilde{x}, \tilde{\lambda}, \varepsilon)$:

$$\begin{aligned} \mathcal{L}(x, \lambda, \varepsilon) =& F(\tilde{x}, \varepsilon) + \tilde{\lambda} G(\tilde{x}, \varepsilon) + \nabla F(\tilde{x}, \varepsilon) dx + \tilde{\lambda} \nabla G(\tilde{x}, \varepsilon) dx + d\lambda G(\tilde{x}, \varepsilon) \\ &+ \frac{1}{2} dx' \Big[\nabla^2 F(\tilde{x}, \varepsilon) + \tilde{\lambda} \nabla^2 G(\tilde{x}, \varepsilon)\Big] dx + d\lambda \nabla G(\tilde{x}, \varepsilon) dx + \mathcal{O}(\varepsilon^3), \end{aligned}$$

where $dx \equiv x - \tilde{x}$ and $d\lambda \equiv \lambda - \tilde{\lambda}$. The first-order optimality condition for $\tilde{x}$ implies

$$\nabla F(\tilde{x}, \varepsilon) + \tilde{\lambda} \nabla G(\tilde{x}, \varepsilon) = 0,$$

while the feasibility of x implies that

$$0 = G(x, \varepsilon) = G(\tilde{x}, \varepsilon) + \nabla G(\tilde{x}, \varepsilon) dx + \mathcal{O}(\varepsilon^2),$$

from which it follows that

$$d\lambda \Big[G(\tilde{x}, \varepsilon) + \nabla G(\tilde{x}, \varepsilon) dx\Big] = \mathcal{O}(\varepsilon^3).$$

Substitute these conditions into the Taylor expansion to obtain the second-order losses relative to the efficient allocation:

$$\begin{aligned} \mathcal{L}(\tilde{x}, \tilde{\lambda}, \varepsilon) - \mathcal{L}(x, \lambda, \varepsilon) &= -\frac{1}{2} dx' \Big[\nabla^2 F(\tilde{x}, \varepsilon) + \tilde{\lambda} \nabla^2 G(\tilde{x}, \varepsilon)\Big] dx + \mathcal{O}(\varepsilon^3) \\ &= -\frac{1}{2} dx' \big[\nabla^2 F(\bar{x}, 0) + \bar{\lambda} \nabla^2 G(\bar{x}, 0)\big] dx + \mathcal{O}(\varepsilon^3), \end{aligned}$$

where we used the fact that $\tilde{\lambda} = \bar{\lambda} + \mathcal{O}(\varepsilon)$, $F(\tilde{x}, \varepsilon) = F(\bar{x}, 0) + \mathcal{O}(\varepsilon)$, $G(\tilde{x}, \varepsilon) = G(\bar{x}, 0) + \mathcal{O}(\varepsilon)$. ■

To apply this result in our setting, write down the Lagrangian for the planner's problem

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \gamma \log C_{Tt} + (1 - \gamma) \left(\log C_{Nt} - \frac{C_{Nt}}{A_t} \right) + \lambda_t \left[B^*_{t-1} + Y_{Tt} - C_{Tt} - \frac{B^*_t}{R^*_t} \right] \Bigg\}.$$

Because the budget constraint is linear in B_t^* and C_{Tt}, the second derivatives are equal to zero and, according to Lemma A2, the second-order approximation to the loss function is given by

$$\tilde{\mathcal{L}}-\mathcal{L}=\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\gamma\left(\frac{C_{Tt}-\tilde{C}_{Tt}}{\bar{C}_T}\right)^2+(1-\gamma)\left(\frac{C_{Nt}-\tilde{C}_{Nt}}{\bar{C}_N}\right)^2\right\}=\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)x_t^2\big],$$

where we used the fact that $z_t=\log(C_{Tt}/\tilde{C}_{Tt})=(C_{Tt}-\tilde{C}_{Tt})/\bar{C}_T+\mathcal{O}(\varepsilon^2)$ and $x_t=\log(C_{Nt}/\tilde{C}_{Nt})=(C_{Nt}-\tilde{C}_{Nt})/\bar{C}_N+\mathcal{O}(\varepsilon^2)$. The resulting expression is consistent with Lemma A1.[53]

A2.2 First-order approximation to the equilibrium system

Non-linear equilibrium system (2)–(3) and (6)–(7), and non-linear wedges:

$$X_t=C_{Nt}/\tilde{C}_{Nt}=C_{Nt}/A_t \quad \text{and} \quad Z_t=C_{Tt}/\tilde{C}_{Tt}.$$

Steady state given by:

$$\bar{B}^*=\bar{F}^*=\bar{N}^*=0, \quad \bar{R}=\bar{R}^*=1/\beta, \quad \bar{C}_T=\bar{Y}_T, \;\; \bar{C}_N=\bar{A},$$

and the associated exchange rates:

$$\bar{\mathcal{E}}=\bar{\mathcal{Q}}=\frac{\gamma}{1-\gamma}\frac{\bar{C}_N}{\bar{C}_T},$$

as well as no steady state wedges, $\bar{X}=\bar{Z}=1$.

Deviations Define for any (endogenous or exogenous) variable Y_t with a non-zero steady state value its log steady-state deviation y_t as:

$$Y_t=\bar{Y}e^{\nu y_t} \quad \text{for} \quad \nu=1,$$

and for net foreign assets B_t^* with a zero steady state value its deviation proportional to tradable steady state output b_t^* as:

$$B_t^*=\bar{Y}_T\nu b_t^* \quad \text{for} \quad \nu=1.$$

For $\nu=0$, we get the steady state values of variables. We take the first order Taylor expansion of the equilibrium system in ν around steady state $\nu=0$ and evaluated at $\nu=1$. The approximate system is linear (scales) in ν, but is not necessarily linear in variables (deviations y_t), as we see below.

[53] In general, Lemma A2 does not require that $\tilde{x}$ is the first best allocation as the system $G(\cdot)=0$ may include constraints due to price stickiness and financial frictions. However, as long as the steady state is efficient, $\bar{\lambda}=0$ for the corresponding constraints, and therefore they do not affect the quadratic loss function.

First best allocation $\{\tilde{C}_{Nt}, \tilde{C}_{Tt}, \tilde{B}_t^*, \tilde{Q}_t\}$ solves $\tilde{C}_{Nt} = A_t$ and:

$$\begin{aligned} \tilde{\mathcal{Q}}_t &= \frac{\gamma}{1-\gamma}\frac{\tilde{C}_{Nt}}{\tilde{C}_{Tt}}, \\ \frac{\tilde{B}_t^*}{R_t^*} - \tilde{B}_{t-1}^* &= Y_{Tt} - \tilde{C}_{Tt}, \\ \beta R_t^* \mathbb{E}_t \frac{\tilde{C}_{Tt}}{\tilde{C}_{Tt+1}} &= 1. \end{aligned}$$

The first order Taylor expansion in ν to this system is given by $\tilde{c}_{Nt} = a_t$ and:

$$\begin{aligned} \tilde{q}_t &= a_t - \tilde{c}_{Tt}, \\ \beta\tilde{b}_t^* - \tilde{b}_{t-1}^* &= y_{Tt} - \tilde{c}_{Tt}, \\ \mathbb{E}_t \Delta\tilde{c}_{Tt+1} &= r_t^*, \end{aligned}$$

where $\{a_t, y_{Tt}, r_t^*\}$ are stochastic shocks (in proportional deviations) determining the dynamics of the first-best allocation.

Proof: Substitute the definitions of variables in terms of ν-deviations into the non-linear system describing the first-best allocation

$$\begin{aligned} \bar{\mathcal{Q}}e^{\nu\tilde{q}_t} &= \frac{\gamma}{1-\gamma}\frac{\bar{C}_N}{\bar{C}_T}e^{\nu(\tilde{c}_{Nt}-\tilde{c}_{Tt})}, \\ \beta e^{-\nu r_t^*}\bar{Y}_T\nu\tilde{b}_t^* - \bar{Y}_T\nu\tilde{b}_{t-1}^* &= \bar{Y}_T e^{\nu y_{Tt}} - \bar{C}_T e^{\nu\tilde{c}_{Tt}}, \\ e^{\nu r_t^*}\mathbb{E}_t e^{-\nu\Delta\tilde{c}_{Tt+1}} &= 1, \end{aligned}$$

where we used the fact that $\bar{R}^* = 1/\beta$. Using the steady state value of $\bar{\mathcal{Q}}$, and the fact that $\tilde{c}_{Nt} = a_t$ (as $\tilde{C}_{Nt} = A_t$), the first equation is immediately log-linear, $\tilde{q}_t = a_t - \tilde{c}_{Tt}$. Dividing the second equation by $\bar{Y}_T$, using the fact that $\bar{C}_T = \bar{Y}_T$, and taking the Taylor expansion, we have:

$$(1 - \nu r_t^* + \mathcal{O}(\nu^2))\nu\beta\tilde{b}_t^* - \nu\tilde{b}_{t-1}^* = \nu y_{Tt} - \nu\tilde{c}_{Tt} + \mathcal{O}(\nu^2),$$

where $\mathcal{O}(\nu^2)$ denotes terms of order ν^2 or higher (around $\nu = 0$). Dividing by ν and eliminating remaining $\mathcal{O}(\nu)$ terms results in the first-order approximate equation. The final equation is expanded as follows:

$$1 = \mathbb{E}_t\{(1 + \nu r_t^* + \mathcal{O}(\nu^2))(1 - \nu\Delta\tilde{c}_{Tt+1} + \mathcal{O}(\nu^2))\} = \mathbb{E}_t\{1 + \nu r_t^* - \nu\Delta\tilde{c}_{Tt+1} + \mathcal{O}(\nu^2)\}.$$

Subtracting 1 on both sides, dividing through by ν, and eliminating the remaining $\mathcal{O}(\nu)$ terms results in the first-order approximate equation. ■

Equilibrium system (2) and (6)–(7) is reproduced here as:

$$\mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{C_{Nt}}{C_{Tt}},$$
$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$$
$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \quad \sigma_t^2 = R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big).$$

Rewrite this system in deviations from the first-best system:

$$\mathcal{E}_t = \tilde{Q}_t \frac{X_t}{Z_t},$$
$$\frac{B_t^* - \tilde{B}_t^*}{R_t^*} - (B_{t-1}^* - \tilde{B}_{t-1}^*) = -(C_{Tt} - \tilde{C}_{Tt}),$$
$$\beta R_t^* \mathbb{E}_t \frac{Z_t \tilde{C}_{Tt}}{Z_{t+1}\tilde{C}_{Tt+1}} - \beta R_t^* \mathbb{E}_t \frac{\tilde{C}_{Tt}}{\tilde{C}_{Tt+1}} = \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \quad \sigma_t^2 = R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big).$$

Define the following additional proportional deviation terms:

$$B_t^* - \tilde{B}_t^* = \bar{Y}_T \nu b_t^*, \quad N_t^* - \tilde{B}_t^* = \bar{Y}_T \nu n_t^*, \quad F_t^* = \bar{Y}_T \nu f_t^*$$

and

$$\omega = \omega_0/\nu^2,$$

for some $\omega_0 \geq 0$, so that ω is the value of risk-aversion at $\nu = 1$, and the value of risk-aversion increases as ν decreases towards 0 (the value of risk-aversion in steady state is irrelevant given the exact absence of risk). Given this definitions, the first-order Taylor approximation in ν to the non-linear equilibrium system around $\nu = 0$ is given by:

$$e_t = \tilde{q}_t + x_t - z_t,$$
$$\beta b_t^* - b_{t-1}^* = -z_t,$$
$$\mathbb{E}_t \Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2 (n_t^* + f_t^* - b_t^*), \quad \bar{\sigma}_t^2 = \mathrm{var}_t(\Delta e_{t+1}).$$

where $\bar{\omega} = \omega_0 \bar{Y}_T/\beta$.

Proof: Following similar steps as above, we substitute the definitions of variables in terms of deviations. The first line immediately results in $e_t = \tilde{q}_t + x_t - z_t$, as the non-linear equation is, in fact, log linear in variables. The second equation yields $\beta b_t^* - b_{t-1}^* = -z_t$ following similar steps as above, and additionally noting that

$$\frac{C_{Tt} - \tilde{C}_{Tt}}{\bar{Y}_T} = e^{\nu c_{Tt}} - e^{\nu \tilde{c}_{Tt}} = \nu z_t + \mathcal{O}(\nu^2),$$

where $z_t = c_{Tt} - \tilde{c}_{Tt}$ by the definition of variables. Finally, the last equilibrium condition is expressed as follows:

$$e^{\nu r_t^*}\mathbb{E}_t\Big[e^{-\nu(\Delta z_{t+1}+\Delta\tilde{c}_{Tt+1})}-e^{-\nu\Delta\tilde{c}_{Tt+1}}\Big]=(\omega_0\bar{Y}_T/\beta)\frac{1}{\nu^2}e^{\nu(2r_t-r_t^*)}\mathrm{var}_t\big(e^{-\nu\Delta e_{t+1}}\big)\nu(b_t^*-n_t^*-f_t^*),$$

after substituting in the expression for σ_t^2 and using $\bar{R}=\bar{R}^*=1/\beta$ to simplify. Dividing both sides by $e^{\nu r_t^*}$, substituting in $\bar{\omega}=\omega_0\bar{Y}_T/\beta$, and taking a first-order Taylor expansion in ν yields:

$$\mathbb{E}_t\big[-\nu\Delta z_{t+1}+\mathcal{O}(\nu^2)\big]=\bar{\omega}(1+2\nu(r_t-r_t^*)+\mathcal{O}(\nu^2))\underbrace{\frac{1}{\nu^2}\mathrm{var}_t\big(1-\nu\Delta e_{t+1}+\mathcal{O}(\nu^2)\big)}_{=\mathrm{var}_t(-\Delta e_{t+1}+\mathcal{O}(\nu))=\mathrm{var}_t(\Delta e_{t+1})+\mathcal{O}(\nu^2)}\nu(b_t^*-n_t^*-f_t^*),$$

Dividing both sides by ν and simplifying:

$$\begin{aligned}-\mathbb{E}_t\Delta z_{t+1}+\mathcal{O}(\nu)&=\bar{\omega}(1+\mathcal{O}(\nu))\big(\mathrm{var}_t(\Delta e_{t+1})+\mathcal{O}(\nu^2)\big)(b_t^*-n_t^*-f_t^*)\\&=\bar{\omega}\mathrm{var}_t(\Delta e_{t+1})(b_t^*-n_t^*-f_t^*)+\mathcal{O}(\nu).\end{aligned}$$

Eliminating the remaining $\mathcal{O}(\nu)$ terms yields the first-order approximate equation in ν, which is however not linear in variables (deviations). ■

We also note that the side equation (3) that determines the path of R_t is approximated in the same way as the other equations of the equilibrium system:

$$r_t=\mathbb{E}_t\Delta c_{Nt+1}=\mathbb{E}_t\Delta x_{t+1}+\mathbb{E}_t\Delta a_{t+1}.$$

Combining the equation for r_t with $\mathbb{E}_t\Delta\tilde{c}_{Tt+1}=r_t^*$ results in the UIP deviation expression (14):

$$\begin{aligned}r_t-r_t^*-\mathbb{E}_t\Delta e_{t+1}&=\mathbb{E}_t\Delta x_{t+1}+\mathbb{E}_t\Delta a_{t+1}-\mathbb{E}_t\Delta\tilde{c}_{Tt+1}-\mathbb{E}_t\Delta e_{t+1}\\&=\mathbb{E}_t\{\Delta x_{t+1}+\Delta\tilde{q}_{t+1}-\Delta e_{t+1}\}=\mathbb{E}_t\Delta z_{t+1},\end{aligned}$$

where we used the facts that $e_t=\tilde{q}_t+x_t-z_t$ and $\tilde{q}_t=a_t-\tilde{c}_{Tt}$. Note: since in our baseline model there is no inflation, $P_{Tt}^*=P_{Nt}=1$, we have that $i_t=r_t$ and $i_t^*=r_t^*$, i.e. nominal and real interest rates coincide.

Lemma A3 *The solution to the approximate equilibrium system characterizes an $\mathcal{O}(\nu)$ accurate dynamics of the non-linear equilibrium system. Furthermore, $\bar{\omega}\bar{\sigma}_t^2-\beta\bar{Y}_T\omega\sigma_t^2=\mathcal{O}(\nu)$, where $\beta\bar{Y}_T$ is the constant of normalization.*

Proof: We first formalize the claim. Consider an exact equilibrium path $\{C_{Nt},C_{Tt},\mathcal{E}_t,B_t^*,\sigma_t^2\}$ that corresponds to policies $\{X_t,F_t^*\}$ and shocks $\{A_t,Y_{Tt},R_t^*,N_t^*\}$, which also determine the first-best allocation $\{\tilde{C}_{Nt},\tilde{C}_{Tt},\tilde{B}_t^*,\tilde{Q}_t\}$. Note that $X_t=C_{Nt}/\tilde{C}_{Nt}$ and $Z_t=C_{Tt}/\tilde{C}_{Tt}$. Define the exact deviations from the steady state $\{\hat{x}_t,\hat{f}_t^*,\hat{z}_t,\hat{e}_t,\hat{b}_t^*,\hat{n}_t^*,\tilde{q}_t\}$ as:

$$X_t=e^{\nu\hat{x}_t},\qquad F_t^*=\bar{Y}_T\nu\hat{f}_t^*,\qquad Z_t=e^{\nu\hat{z}_t},\qquad \mathcal{E}_t=\bar{\mathcal{E}}e^{\nu\hat{e}_t},\qquad B_t^*-\tilde{B}_t^*=\bar{Y}_T\nu\hat{b}_t^*,$$

$$N_t^*-\tilde{B}_t^*=\bar{Y}_T\nu\hat{n}_t^*\qquad\text{and}\qquad\tilde{Q}_t=\bar{\mathcal{Q}}e^{\nu\tilde{q}_t}\qquad\text{for}\qquad\nu=1.$$

We define similarly $\{\tilde{c}_{Tt},\hat{r}_t,\hat{r}_t^*,\hat{y}_{Tt},\hat{a}_t\}$.

Consider now an approximate equilibrium path $\{z_t, e_t, b_t^*, \bar{\sigma}_t^2\}$ that emerges as a result of policies $\{\hat{x}_t, \hat{f}_t^*\}$ in response to shocks $\{\hat{n}_t^*, \tilde{q}_t\}$. Then:

$$\{z_t, e_t, b_t^*, \bar{\omega}\bar{\sigma}_t^2\} - \{\hat{z}_t, \hat{e}_t, \hat{b}_t^*, \beta\bar{Y}_T\omega\sigma_t^2\} = \mathcal{O}(\nu),$$

where $\bar{\sigma}_t^2 = \mathrm{var}_t(\Delta e_{t+1})$, $\sigma_t^2 = \beta^{-2}e^{2\nu\hat{r}_t} \cdot \mathrm{var}_t(e^{-\nu\Delta\hat{e}_{t+1}})$, and $\omega = \omega_0/\nu^2$ in the exact system and $\bar{\omega} = \omega_0\bar{Y}_T/\beta$ in the approximate system for some constant ω_0.

The proof of this formal claim follows from the first-order Taylor expansion of the equilibrium system in exact deviations from the first best $\{\hat{z}_t, \hat{e}_t, \hat{b}_t^*\}$, described above, which we rewrite here as:

$$\begin{aligned}
\nu\hat{e}_t &= \nu(\tilde{q}_t + \hat{x}_t - \hat{z}_t), \\
\beta e^{\nu\hat{r}_t^*}\nu\hat{b}_t^* - \nu\hat{b}_{t-1}^* &= -\big(e^{\nu(\hat{z}_t + \tilde{c}_{Tt})} - e^{\nu\tilde{c}_{Tt}}\big), \\
\mathbb{E}_t e^{-\nu(\Delta\hat{z}_{t+1} + \Delta\tilde{c}_{Tt+1})} - \mathbb{E}_t e^{-\nu\Delta\tilde{c}_{Tt+1}} &= \frac{\omega_0\bar{Y}_T/\beta}{\nu^2}e^{2\nu(\hat{r}_t - \hat{r}_t^*)}\mathrm{var}_t\big(e^{-\nu\Delta\hat{e}_{t+1}}\big)\nu(\hat{b}_t^* - \hat{n}_t^* - \hat{f}_t^*).
\end{aligned}$$

The first-order Taylor expansion of this exact system is:

$$\begin{aligned}
\hat{e}_t &= \tilde{q}_t + \hat{x}_t - \hat{z}_t, \\
\beta\hat{b}_t^* - \hat{b}_{t-1}^* &= -\hat{z}_t + \mathcal{O}(\nu), \\
\mathbb{E}_t\Delta\hat{z}_{t+1} &= \bar{\omega}\mathrm{var}_t(\Delta\hat{e}_{t+1})(\hat{n}_t^* + \hat{f}_t^* - \hat{b}_t^*) + \mathcal{O}(\nu),
\end{aligned}$$

while the approximate system for $\{z_t, e_t, b_t^*\}$ is:

$$\begin{aligned}
e_t &= \tilde{q}_t + \hat{x}_t - z_t, \\
\beta b_t^* - b_{t-1}^* &= -z_t, \\
\mathbb{E}_t\Delta z_{t+1} &= \bar{\omega}\mathrm{var}_t(\Delta e_{t+1})(\hat{n}_t^* + \hat{f}_t^* - \hat{b}_t^*),
\end{aligned}$$

Therefore, the difference between the exact solution $\{\hat{z}_t, \hat{e}_t, \hat{b}_t^*\}$ and the approximate solution $\{z_t, e_t, b_t^*\}$ vanishes with $\mathcal{O}(\nu)$. Furthermore:

$$\omega\sigma_t^2 = \frac{\omega_0}{\nu^2}\beta^{-2}e^{2\nu\hat{r}_t}\mathrm{var}_{t-1}\big(e^{-\nu\Delta\hat{e}_{t+1}}\big) = \frac{\omega_0}{\beta^2}\mathrm{var}_{t-1}(\Delta\hat{e}_{t+1}) + \mathcal{O}(\nu) = \frac{1}{\beta\bar{Y}_T}\bar{\omega}\bar{\sigma}_t^2 + \mathcal{O}(\nu),$$

where the last equality holds because $\mathrm{var}_t(\Delta\hat{e}_{t+1}) - \mathrm{var}_t(\Delta e_{t+1}) = \mathcal{O}(\nu^2)$ as $\{\hat{e}_t\} - \{e_t\} = \mathcal{O}(\nu)$, and we used $\bar{\omega} = \omega_0\bar{Y}_T/\beta$. ■

A2.3 Optimal policies

Set up a Lagrangian for the policy problem (15) for any given path of $\{f_t^*\}$:

$$\ell_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \bigg[\frac{1}{2}\big(\gamma z_t^2 + (1-\gamma)x_t^2\big) - \gamma\lambda_t\big(b_{t-1}^* - z_t - \beta b_t^*\big) \tag{A7}$$
$$- \gamma\mu_t\Big(\mathbb{E}_t\Delta z_{t+1} - \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*)\Big)$$
$$- \delta_t\Big(\bar{\sigma}_t^2 - \mathbb{E}_t(\tilde{q}_{t+1} + x_{t+1} - z_{t+1})^2 + \big(\mathbb{E}_t(\tilde{q}_{t+1} + x_{t+1} - z_{t+1})\big)^2\Big)\bigg],$$

where we substituted in the expression for $e_t = \tilde{q}_t + x_t - z_t$ and replaced $\bar{\sigma}_t^2 = \mathrm{var}_t(\Delta e_{t+1}) = \mathbb{E}_t e_{t+1}^2 - (\mathbb{E}_t e_{t+1})^2$. Note the analogy with the non-linear Lagrangian $\mathcal{L}_0$ in Appendix A1; the fact that we have negatives in front of the constraints in the approximate problem reflects the fact that we are minimizing welfare loss, in contrast to maximizing welfare in the exact problem.

The optimality conditions with respect to $\{x_t, z_t, b_t^*, \bar{\sigma}_t^2\}$ are:

$$\begin{aligned}
0 &= (1-\gamma)x_t + 2\beta^{-1}\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t),\\
0 &= \gamma z_t + \gamma\lambda_t + (\gamma\mu_t - \beta^{-1}\gamma\mu_{t-1}) - 2\beta^{-1}\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t),\\
0 &= \beta(\gamma\lambda_t - \mathbb{E}_t\gamma\lambda_{t+1}) - \gamma\mu_t\bar{\omega}\bar{\sigma}_t^2,\\
0 &= \gamma\mu_t\bar{\omega}(n_t^* + f_t^* - b_t^*) - \delta_t.
\end{aligned}$$

where we define $\delta_{-1} = \mu_{-1} = 0$. After simplification:

$$\begin{aligned}
\beta(1-\gamma)x_t &= -2\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t),\\
\gamma z_t + (1-\gamma)x_t &= -\gamma\lambda_t - \gamma(\mu_t - \beta^{-1}\mu_{t-1}),\\
\lambda_t - \mathbb{E}_t\lambda_{t+1} &= \beta^{-1}\mu_t\bar{\omega}\bar{\sigma}_t^2,\\
\delta_t &= \gamma\mu_t\bar{\omega}(n_t^* + f_t^* - b_t^*).
\end{aligned}$$

These optimality conditions, together with the constraints in the policy problem (A7) characterize the optimal monetary policy $\{x_t\}$ for a given path of FXI $\{f_t^*\}$ and the associated equilibrium allocation.

Lemma A4 *The optimality conditions for the approximate policy problem* (A7) *correspond to the first-order Taylor expansion (in ν around $\nu = 0$) of the non-linear optimality conditions for the exact policy problem* (A1′).

Proof: Given Lemma A3, it remains to show that the first-order Taylor expansion of the exact optimality conditions (A3)–(A6) results in the same system of equations as the first order conditions to the approximate problem (A7) given above.

In addition to the definitions of ν-deviations of variables in Appendix A2, we define the deviations for multipliers $\{M_t, \Lambda_t, D_t\}$ in the Lagrangian $\mathcal{L}_0$ for the exact policy problem (A1′):

$$\Lambda_t = \bar{\Lambda}e^{\nu\lambda_t}, \qquad M_t = \bar{M}\nu\mu_t, \qquad D_t' = \bar{D}'\delta_t,$$

where $\bar{\Lambda} = \gamma/\bar{C}_T = \gamma/\bar{Y}_T$, $\bar{M} = \gamma$, and $\bar{D}' = \bar{\mathcal{E}}^2$ are proportional scalers. Note that M_t and D'_t are equal to zero in a zero-NFA steady state; furthermore, there is only a zero-order component of D'_t, which can be verified by generalizing $D'_t = \bar{D}'\delta_t + \nu d_t + \mathcal{O}(\nu^2)$ and showing that $d_t \equiv 0$ using our approximation below.[54]

Consider first the expansion of (A3)–(A6):

$$\bar{\Lambda}\big(e^{\nu\lambda_t} - e^{\nu r_t^*}\mathbb{E}_t e^{\nu\lambda_{t+1}}\big) = \bar{M}\nu\mu_t\frac{\omega_0}{\nu^2}\beta^{-2}e^{2\nu r_t}\mathrm{var}_t\big(e^{-\nu\Delta e_{t+1}}\big),$$
$$\bar{D}'\delta_t = \bar{M}\nu\mu_t\frac{\omega_0/\beta}{\nu^2}\bar{\mathcal{E}}^2 e^{2\nu(e_t+r_t)-\nu r_t^*}\bar{Y}_T\nu(n_t^* + f_t^* - b_t^*),$$
$$\beta(1-\gamma)(e^{\nu x_t} - 1) = \frac{2\bar{D}'}{\bar{\mathcal{E}}^2}\delta_{t-1}\Big[e^{-2\nu e_t} - e^{-\nu e_t}\mathbb{E}_{t-1}e^{-\nu e_t} - \frac{e^{-\nu c_{Nt}}}{\mathbb{E}_{t-1}e^{-\nu c_{Nt}}}\Big(\mathbb{E}_{t-1}e^{-2\nu e_t} - (\mathbb{E}_{t-1}e^{-\nu e_t})^2\Big)\Big],$$
$$\gamma\big(1 - e^{\nu(\lambda_t + c_{Tt})}\big) - (1-\gamma)(e^{\nu x_t} - 1) = \bar{M}\nu\mu_t\Big(e^{\nu r_t^*}\mathbb{E}_t e^{-\Delta c_{Tt+1}} + 2\omega\sigma_t^2\beta e^{-\nu r_t^*}\nu(n_t^* + f_t^* - b_t^*)\Big)$$
$$- \beta^{-1}\bar{M}\nu\mu_{t-1}\Big(e^{\nu r_{t-1}^* - \Delta c_{Tt}} + 2\omega\sigma_{t-1}^2\beta e^{-\nu r_{t-1}^*}\nu(n_t^* + f_t^* - b_t^*)\frac{e^{-\nu c_{Nt}}}{\mathbb{E}_{t-1}e^{-\nu c_{Nt}}}\Big),$$

where we used $\omega = \omega_0/\nu$ and $\Gamma_t = 1/C_{Nt} = e^{-\nu c_{Nt}}/\bar{A}$. We take a first order Taylor expansion in ν around $\nu = 0$:

$$\nu\lambda_t - \nu r_t^* - \mathbb{E}_t\nu\lambda_{t+1} + \mathcal{O}(\nu^2) = \beta^{-1}\mu_t\nu(1 + 2\nu r_t + \mathcal{O}(\nu^2))\big(\mathrm{var}_t(\Delta e_{t+1}) + \mathcal{O}(\nu^2)\big),$$
$$\delta_t = \gamma\mu_t\bar{\omega}\big(1 + 2\nu(e_t + r_t) - \nu r_t^* + \mathcal{O}(\nu^2)\big)(n_t^* + f_t^* - b_t^*),$$
$$\beta(1-\gamma)\nu x_t = 2\delta_{t-1}\big[-\nu(e_t - \mathbb{E}_{t-1}e_t) + \mathcal{O}(\nu^2) - \big(1 + O(\nu)\big)\mathcal{O}(\nu^2)\big],$$
$$-\gamma\nu(\lambda_t + c_{Tt}) + \mathcal{O}(\nu) - (1-\gamma)\nu x_t = \gamma\nu\mu_t\big(1 + \mathcal{O}(\nu)\big) - \beta^{-1}\gamma\nu\mu_{t-1}\big(1 + \mathcal{O}(\nu)\big),$$

where we used the definitions of $(\bar{\Lambda}, \bar{M}, \bar{D}')$ and $\bar{\omega} = \omega_0\bar{Y}_T/\beta$, and the result in Lemma A3 that $\omega\sigma_t^2 - \bar{\omega}\bar{\sigma}_t^2 = \mathcal{O}(\nu)$ and $\nu\bar{\omega}\bar{\sigma}_t^2 = \mathcal{O}(\nu)$. Dividing all equations (except for the second line) by ν and grouping together the remaining higher order terms, we obtain:

$$\bar{\lambda}_t - \mathbb{E}_t\bar{\lambda}_{t+1} = \beta^{-1}\mu_t\bar{\omega}\bar{\sigma}_t^2 + \mathcal{O}(\nu),$$
$$\delta_t = \gamma\mu_t\bar{\omega}(n_t^* + f_t^* - b_t^*) + \mathcal{O}(\nu),$$
$$\beta(1-\gamma)x_t = -2\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t) + \mathcal{O}(\nu),$$
$$\gamma z_t + (1-\gamma)x_t = -\gamma\bar{\lambda}_t - (\gamma\mu_t - \beta^{-1}\gamma\mu_{t-1}) + \mathcal{O}(\nu),$$

where $\bar{\sigma}_t^2 = \mathrm{var}_t(\Delta e_{t+1})$, and we used the optimality condition for the first best tradable consumption, $r_t^* = \mathbb{E}_t\Delta\tilde{c}_{Tt+1}$, the definition of $z_t = c_{Tt} - \tilde{c}_{Tt}$, and additionally denoted with $\bar{\lambda}_t = \lambda_t + \tilde{c}_{Tt}$. Dropping the higher order terms $\mathcal{O}(\nu)$, this system corresponds to the optimality conditions of the approximate problem. ■

[54] Note from the solution that δ_{t-1} is the slope of the policy rule, $\beta(1-\gamma)x_t = -2\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t)$ and, just like $\bar{\omega}\bar{\sigma}_t^2$, it does not scale with ν, while other deviations (in particular, those of X_t and $\mathcal{E}_t$) scale proportionally with ν. In other words, the risk premium and the slope of the optimal policy are zero order in ν.

A3 Derivations and Proofs for Section 3

Proof of Theorem 1 Consider the optimality conditions for the approximate policy problem (A7) derived in Appendix A2.3. In particular, the first and the last optimality conditions (with respect to x_t and σ_t^2) are given by:

$$\beta(1-\gamma)x_t = -2\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t),$$
$$\delta_t = \gamma\mu_t\bar{\omega}(n_t^* + f_t^* - b_t^*),$$

and we write the former condition for $t+1$:

$$x_{t+1} = -\frac{2}{\beta(1-\gamma)}\delta_t(e_t - \mathbb{E}_{t-1}e_t).$$

Therefore, a rescaled $\delta_t := 2\delta_t/[\beta(1-\gamma)]$ is the monetary policy lean against exchange rate surprises at $t+1$, and since it is predetermined at t, we have $\mathbb{E}_t x_{t+1} = 0$. Additionally rescaling $\mu_t := \beta^{-1}\mu_t$ the Lagrange multiplier on the risk-sharing constraint $\mathbb{E}_t\Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*)$ completes the proof. ■

Proof of Proposition 1 Consider the approximate policy problem (A7) where the choice of FXI f_t^* is unconstrained, and therefore we have an additional optimality condition:

$$\mu_t = 0 \quad \text{for all } t.$$

From the other optimality conditions, we have $x_t = \delta_t = 0$ for all $t \geq 0$, as well as:

$$\mathbb{E}_t\Delta\lambda_{t+1} = -\mathbb{E}_t\Delta z_{t+1} = 0,$$

which together with the budget constraint implies $z_t = b_t^* = 0$ for all t as the unique solution. Consequently, $e_t = \tilde{q}_t$, and $\bar{\sigma}_t^2 = \mathrm{var}_t(\Delta\tilde{q}_{t+1})$. Finally, $\mathbb{E}_t\Delta z_{t+1} = 0$ requires:

$$\bar{\omega}\bar{\sigma}_t(n_t^* + f_t^* - b_t^*) = 0,$$

and thus generically FXI must satisfy:

$$f_t^* = b_t^* - n_t^* = -n_t^*.$$

Note that $f_t^* = -n_t^*$ guarantees $z_t = b_t^* = 0$ as the unique equilibrium, as the non-linear system:

$$\mathbb{E}_t\Delta z_{t+1} = -\bar{\omega}\bar{\sigma}_t^2 b_t^*, \qquad \bar{\sigma}_t^2 = \mathrm{var}_t(\tilde{q}_{t+1} - z_{t+1}),$$
$$\beta b_t^* - b_{t-1}^* = -z_t$$

has a unique stable solution $z_t = b_t^* = 0$.

Lastly, consider the discretionary solution with the planner choosing the optimal policy as a function of natural state variables $(b_{t-1}^*, \tilde{q}_t, n_t^*)$. This implies that private agents form their expectations about future policies $z_{t+1} = z(b_t^*, \tilde{q}_{t+1}, n_{t+1}^*)$ and $x_{t+1} = x(b_t^*, \tilde{q}_{t+1}, n_{t+1}^*)$. The only way the planner can credibly manipulate the beliefs in period t in the absence of commitment is by changing the future state b_t^*. The resulting policy problem corresponds to finding the Markov perfect equilibrium:

$$\begin{aligned} V(b^*, \tilde{q}, n^*) = \min_{\{z,x,f^*,b^{*\prime},\sigma^2\}} \quad & \frac{1}{2}\big[\gamma z^2 + (1-\gamma)x^2\big] + \beta\mathbb{E}\big[V(b^{*\prime}, \tilde{q}', n^{*\prime})|\tilde{q}, n^*\big] \\ \text{subject to} \quad & \beta b^{*\prime} = b^* - z, \\ & \mathbb{E}\big[z(b^{*\prime}, \tilde{q}', n^{*\prime})|\tilde{q}, n^*\big] = z + \bar{\omega}\sigma^2(n^* + f^* - b^*), \\ & \sigma^2 = \text{var}\Big(\tilde{q}' + x(b^{*\prime}, \tilde{q}', n^{*\prime}) - z(b^{*\prime}, \tilde{q}', n^{*\prime})|\tilde{q}, n^*\Big), \end{aligned} \tag{A8}$$

where functions $x(\cdot)$ and $z(\cdot)$ should be consistent with the solution to this policy problem. Following the primal approach, observe that given a free choice of f^*, the latter two constraints do not bind. It follows that the $x(b^*, \tilde{q}, n^*) = 0$ and problem reduces to

$$V(b^*, \tilde{q}, n^*) = \min_{b^{*\prime}} \quad \frac{\gamma}{2}\big(b^* - \beta b^{*\prime}\big)^2 + \beta\mathbb{E}V(b^{*\prime}, \tilde{q}', n^{*\prime}).$$

A combination of the first-order and envelope conditions implies that $z(b^*, \tilde{q}, n^*) = (1-\beta)b^*$. It follows from $b_{-1}^* = 0$ that $z_t = b_t^* = 0$ and the discretionary policy implements the same allocation as the optimal policy under commitment. This allocation is also supported by the same prices, FXI and monetary policy. ■

Proof of Proposition 2 Consider the case with $\tilde{q}_t \equiv \bar{q} = 0$ for all t, the latter equality without loss of generality given our notation in terms of deviations. Then the equilibrium system becomes:

$$\begin{aligned} e_t &= x_t - z_t, \\ \beta b_t^* - b_{t-1}^* &= -z_t, \\ \mathbb{E}_t \Delta z_{t+1} &= \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*), \quad \bar{\sigma}_t^2 = \text{var}_t(\Delta e_{t+1}), \end{aligned}$$

and it is consistent with a $x_t = z_t = b_t^* = \sigma_t^2 = 0$ equilibrium for all t independently of $\{n_t^*, f_t^*\}$. Since this corresponds to the first best, achieving the global minimum of the welfare loss objective, it is the solution to the optimal policy problem (A7). Indeed, with $\lambda_t = \mu_t = \delta_t = 0$ for all t, all optimality conditions of (A7) are satisfied, and in particular (16) in Theorem 1 holds irrespective of $\{n_t^*, f_t^*\}$.

In general, when $\tilde{q}_t = 0$ and $x_t = 0$, there exist other equilibria with $\sigma_t^2 > 0$, such that:

$$\begin{aligned} \beta b_t^* - b_{t-1}^* &= -z_t, \\ \mathbb{E}_t \Delta z_{t+1} &= \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*), \quad \bar{\sigma}_t^2 = \text{var}_t(z_{t+1}), \end{aligned}$$

and thus output gap targeting, $x_t = 0$, does not guarantee $z_t = b_t^* = 0$ as the unique equilibrium. In contrast, a policy rule that targets the exchange rate, $x_t = -\delta(e_t - \mathbb{E}_{t-1}e_t)$ with $\delta \to \infty$, ensures $x_t = z_t = b_t^* = \sigma_t^2 = 0$ as the only equilibrium outcome. Indeed, such a rule implies, using $e_t = x_t - z_t$, that $x_t = -\frac{\delta}{1+\delta}(z_t - \mathbb{E}_{t-1}z_t) = -(z_t - \mathbb{E}_{t-1}z_t)$ as $\delta \to \infty$. This, in turn, means $e_t = \mathbb{E}_{t-1}e_t$ and

$\sigma_t^2 = 0$, which ensures $z_t = b_t^* = 0$, and hence $x_t = 0$, irrespectively of $\{n_t^*, f_t^*\}$. ■

Proof of Proposition 3 Without commitment, the planner solves problem (A8) with an additional constraint that the path of f_t^* is exogenous. Given the values of state variables $(b^*, \tilde{q}, n^*)$, there are three endogenous variables $(b^{*\prime}, z, \sigma^2)$ in the three constraints of the problem. It follows that the choice of x affects the nominal exchange rate $e = \tilde{q} + x - z$, but not capital flows $n^* + f^* - b^*$ or UIP deviations $\mathbb{E}\big[z(b^{*\prime}, \tilde{q}', n^{*\prime})|\tilde{q}, n^*\big] - z$. Neither does it change the risk-sharing wedge z or the continuation value $V(b^{*\prime}, \tilde{q}', n^{*\prime})$, which implies that it is optimal for the monetary policy to set $x = 0$. ■

Proof of Proposition 4 Rescale $\mu_t := \beta^{-1}\mu_t$ for this proof. Let f_t^* be chosen optimally at all $t \geq 1$, but fixed exogenously at f_0^* at $t = 0$. This generalizes to any $t \geq 0$ (see a general solution following the proof of Proposition 5 below). Note that an unconstrained choice of f_t^* for $t > 0$ implies $\mu_t = 0$, which further implies $\delta_t = 0$, $x_{t+1} = 0$ and $\mathbb{E}_t\lambda_{t+1} = \lambda_t$. We then have (using optimality conditions for (A7) and Theorem 1):

$$\mu_t = 0 \quad \forall t \geq 1 \quad \Rightarrow \quad x_t = 0 \quad \forall t \neq 1, \quad \mathbb{E}_0 x_1 = 0 \quad \text{and} \quad (1-\gamma)x_1 = 2\mu_0\bar{\omega}(n_0^* + f_0^* - b_0^*)(e_1 - \mathbb{E}_0 e_1),$$

as well as $\mathbb{E}_t\Delta\lambda_{t+1} = 0$ for all $t \geq 1$ and $\beta b_0^* = -z_0$. Furthermore, $\gamma z_t = \lambda_t$ for all $t \geq 2$ (since $x_t = \mu_t = 0$ for all $t \geq 1$), and therefore:

$$\forall t \geq 2: \quad \gamma\mathbb{E}_t\Delta z_{t+1} = \mathbb{E}_t\Delta\lambda_{t+1} = 0 \quad \Rightarrow \quad f_t^* = b_t^* - n_t^*.$$

We use $\gamma z_t + (1-\gamma)x_t = \lambda_t + \beta\mu_t - \mu_{t-1}$ for $t = 0, 1$ together with $\mathbb{E}_0\Delta\lambda_1 = -\bar{\omega}\bar{\sigma}_0^2\mu_0$ and $\mathbb{E}_0\Delta z_1 = \bar{\omega}\bar{\sigma}_0^2(n_0^* + f_0^* - b_0^*)$ to solve for:

$$\begin{aligned}
\mu_0 &= \frac{\gamma\bar{\omega}\bar{\sigma}_0^2}{1+\beta+\bar{\omega}\bar{\sigma}_0^2}(n_0^* + f_0^* - b_0^*),\\
x_1 &= -\frac{2\bar{\omega}\gamma}{1-\gamma}\frac{\bar{\omega}\bar{\sigma}_0^2}{1+\beta+\bar{\omega}\bar{\sigma}_0^2}(n_0^* + f_0^* - b_0^*)^2(e_1 - \mathbb{E}_0 e_1),\\
e_1 - \mathbb{E}_0 e_1 &= \frac{(\tilde{q}_1 - z_1) - \mathbb{E}_0(\tilde{q}_1 - z_1)}{1 + \frac{2\bar{\omega}\gamma}{1-\gamma}\frac{\bar{\omega}\bar{\sigma}_0^2}{1+\beta+\bar{\omega}\bar{\sigma}_0^2}(n_0^* + f_0^* - b_0^*)^2},
\end{aligned}$$

where $\bar{\sigma}_0^2 = \mathrm{var}_0(e_1)$ and we used $e_1 - \mathbb{E}_0 e_1 = x_1 + (\tilde{q}_1 - \tilde{z}_1) - \mathbb{E}_0(\tilde{q}_1 - z_1)$. This provides the equation used in Proposition 4 as functions of endogenous b_0^* and $\bar{\sigma}_0^2$, which we characterize next.

We use optimality condition $\gamma z_t + (1-\gamma)x_t = \lambda_t + \beta\mu_t - \mu_{t-1}$ in difference at $t = 2$:

$$\gamma\Delta z_2 = (1-\gamma)x_1 + \mu_0 = -\frac{\gamma\bar{\omega}\bar{\sigma}_0^2}{1+\beta+\bar{\omega}\bar{\sigma}_0^2}(n_0^* + f_0^* - b_0^*)\left[1 + 2\bar{\omega}(n_0^* + f_0^* - b_0^*)(e_1 - \mathbb{E}_0 e_1)\right]$$

because $\Delta\lambda_2 = \mathbb{E}_1\Delta\lambda_2 = 0$ as there is no uncertainty in (x_t, z_t, b_t^*) after $t = 1$. We solve for f_1^* from:

$$\bar{\omega}\bar{\sigma}_1^2(n_1^* + f_1^* - b_1^*) = \mathbb{E}_1\Delta z_2 = \Delta z_2 = -\frac{\bar{\omega}\bar{\sigma}_0^2}{1+\beta+\bar{\omega}\bar{\sigma}_0^2}(n_0^* + f_0^* - b_0^*)\left[1 + 2\bar{\omega}(n_0^* + f_0^* - b_0^*)(e_1 - \mathbb{E}_0 e_1)\right],$$

where $\bar{\sigma}_1^2 = \mathrm{var}_1(e_2) = \mathrm{var}_1(\tilde{q}_2) = \sigma_{\tilde{q}}^2$. Note that $\beta b_1^* = b_0^* - z_1 = -\beta^{-1} z_0 - z_1$, and then $\beta b_t^* = b_{t-1}^* - z_2$ for $t \geq 2$.

Finally, we close by solving for (z_0, z_1, z_2) from the intertemporal budget constraint

$$z_0 + \beta z_1 + \frac{\beta^2}{1-\beta} z_2 = 0, \qquad z_t = z_2 \quad \forall t \geq 2,$$

and given solution for Δz_2 and $\mathbb{E}_0 \Delta z_1 = \bar{\omega}\bar{\sigma}_0^2(n_0^* + f_0^* - b_0^*)$, and hence we have:

$$b_0^* = -\beta^{-1} z_0 = \mathbb{E}_0[\Delta z_1 + \beta \Delta z_2] = \frac{1+\bar{\omega}\bar{\sigma}_0^2}{1+\beta+\bar{\omega}\bar{\sigma}_0^2}\bar{\omega}\bar{\sigma}_0^2(n_0^* + f_0^* - b_0^*) \quad \Rightarrow \quad b_0^* = \frac{(1+\bar{\omega}\bar{\sigma}_0^2)\bar{\omega}\bar{\sigma}_0^2}{\beta + (1+\bar{\omega}\bar{\sigma}_0^2)^2}(n_0^* + f_0^*)$$

and

$$n_0^* + f_0^* - b_0^* = \frac{1+\beta+\bar{\omega}\bar{\sigma}_0^2}{\beta+(1+\bar{\omega}\bar{\sigma}_0^2)^2}(n_0^* + f_0^*).$$

Then we solve:

$$z_1 = (1-\beta) b_0^* - \beta \Delta z_2 = \frac{\bar{\omega}\bar{\sigma}_0^2 (n_0^* + f_0^*)}{\beta + (1+\bar{\omega}\bar{\sigma}_0^2)^2}\left[(1-\beta)(1+\bar{\omega}\bar{\sigma}_0^2) + \beta\left(1 + 2\bar{\omega}\frac{1+\beta+\bar{\omega}\bar{\sigma}_0^2}{\beta+(1+\bar{\omega}\bar{\sigma}_0^2)^2}(n_0^* + f_0^*)\tilde{e}_1\right)\right],$$

where $\tilde{e}_1 \equiv e_1 - \mathbb{E}_0 e_1$, so that:

$$\begin{aligned}
z_1 - \mathbb{E}_0 z_1 &= \frac{2\beta\bar{\omega}^2\bar{\sigma}_0^2(1+\beta+\bar{\omega}\bar{\sigma}_0^2)}{[\beta+(1+\bar{\omega}\bar{\sigma}_0^2)^2]^2}(n_0^* + f_0^*)^2 \tilde{e}_1, \\
x_1 &= -\frac{2\gamma}{1-\gamma}\frac{\bar{\omega}^2\bar{\sigma}_0^2(1+\beta+\bar{\omega}\bar{\sigma}_0^2)}{[\beta+(1+\bar{\omega}\bar{\sigma}_0^2)^2]^2}(n_0^* + f_t^*)^2 \tilde{e}_1, \\
\tilde{e}_1 &= \frac{\tilde{q}_1 - \mathbb{E}_0 \tilde{q}_1}{1 + [\gamma + \beta(1-\gamma)]\frac{2\bar{\omega}^2\bar{\sigma}_0^2(1+\beta+\bar{\omega}\bar{\sigma}_0^2)}{(1-\gamma)[\beta+(1+\bar{\omega}\bar{\sigma}_0^2)^2]^2}(n_0^* + f_0^*)^2}
\end{aligned}$$

so that $\bar{\sigma}_0^2$ solves the fixed point:

$$\bar{\sigma}_0^2 = \left(1 + [\gamma + \beta(1-\gamma)]\frac{2\bar{\omega}^2\bar{\sigma}_0^2(1+\beta+\bar{\omega}\bar{\sigma}_0^2)}{(1-\gamma)[\beta+(1+\bar{\omega}\bar{\sigma}_0^2)^2]^2}(n_0^* + f_0^*)^2\right)^{-2} \sigma_{\tilde{q}}^2,$$

completing the characterization. ■

Proof of Theorem 2 For any given path $\{x_t\}$, consider the optimality conditions for z_t, b_t^* and $\bar{\sigma}_t^2$ in problem (A7):

$$\begin{aligned}
0 &= z_t + \lambda_t + (\beta\mu_t - \mu_{t-1}) - 2\delta_{t-1}(e_t - \mathbb{E}_{t-1} e_t), \\
\mathbb{E}_t \lambda_{t+1} - \lambda_t &= -\mu_t \bar{\omega}\bar{\sigma}_t^2, \\
\delta_t &= \mu_t \bar{\omega}(n_t^* + f_t^* - b_t^*).
\end{aligned}$$

Writing the first condition in expected first difference at t and using the other two conditions to substitute out δ_t and λ_t, we have:

$$\big(1+\beta+\bar{\omega}\sigma_t^2\big)\mu_t-\beta\mathbb{E}_t\mu_{t+1}-\big[1+2\bar{\omega}(n_{t-1}^*+f_{t-1}^*-b_{t-1}^*)(e_t-\mathbb{E}_{t-1}e_t)\big]\mu_{t-1}=\mathbb{E}_t\Delta z_{t+1},$$

which corresponds to equation (18). The risk-sharing condition requires additionally that $\mathbb{E}_t\Delta z_{t+1}=\bar{\omega}\sigma_t^2(n_t^*+f_t^*-b_t^*)$. If the constraint $f_t^*\in\mathcal{F}_t$ is binding, this determines the value of $\mathbb{E}_t\Delta z_{t+1}$ and, thus, μ_t. If f_t^* is unconstrained, then $\mu_t=0$. ■

Proof of Proposition 5 We follow Marcet and Marimon (2019) and rewrite the planner's problem (A7) in a recursive form. To this end, let $z_t^e=\mathbb{E}_t z_{t+1}$ denote agents' expectations and use optimal policy rule from Theorem 1:

$$x_{t+1}=-\bar{\delta}_t(\tilde{q}_{t+1}-z_{t+1}+z_t^e),\qquad \bar{\delta}_t\equiv\frac{\delta_t}{1+\delta_t},$$

where for brevity we use $\tilde{q}_{t+1}$ to denote deviations of the natural real exchange rate from conditional expectation in period t. Take a timeless perspective with the planner committing to state-contingent policies before the realization of shocks and rewrite the corresponding Lagrangian as follows:

$$\begin{aligned}\ell_0=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{&\frac{1}{2}\Big[\gamma z_t^2+(1-\gamma)\bar{\delta}_{t-1}^2(\tilde{q}_t-z_t)^2-(1-\gamma)\bar{\delta}_{t-1}^2(z_{t-1}^e)^2\Big]-\gamma\mu_t\Big[z_t^e-z_t-\bar{\omega}\bar{\sigma}_t^2(n_t^*+f_t^*-b_t^*)\Big]\\
&+\phi_t\Big[(1-\bar{\delta}_t)^2\mathbb{E}_t(\tilde{q}_{t+1}-z_{t+1})^2-(1-\bar{\delta}_t)^2(z_t^e)^2-\bar{\sigma}_t^2\Big]+\gamma\lambda_t(\beta b_t^*-b_{t-1}^*+z_t)+\xi_t(z_t^e-\mathbb{E}_t z_{t+1})\Bigg\}\\
=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{&\frac{1}{2}\Big[\gamma z_t^2+(1-\gamma)\bar{\delta}_{t-1}^2(\tilde{q}_t-z_t)^2-\beta(1-\gamma)\bar{\delta}_t^2(z_t^e)^2\Big]-\gamma\mu_t\Big[z_t^e-z_t-\bar{\omega}\bar{\sigma}_t^2(n_t^*+f_t^*-b_t^*)\Big]\\
&+\frac{\phi_{t-1}}{\beta}(1-\bar{\delta}_{t-1})^2(\tilde{q}_t-z_t)^2-\phi_t(1-\bar{\delta}_t)^2(z_t^e)^2-\phi_t\bar{\sigma}_t^2+\gamma\lambda_t(\beta b_t^*-b_{t-1}^*+z_t)+\xi_t z_t^e-\frac{\xi_{t-1}}{\beta}z_t\Bigg\}.\end{aligned}$$

The first-order condition with respect to $\bar{\delta}_t$ implies that

$$\Big[\beta(1-\gamma)\bar{\delta}_t-2\phi_t(1-\bar{\delta}_t)\Big]\Big[\mathbb{E}_t(\tilde{q}_{t+1}-z_{t+1})^2-(z_t^e)^2\Big]=0$$

and hence, the Lagrange multiplier ϕ_t is proportionate to $\delta_t=\frac{\bar{\delta}_t}{1-\bar{\delta}_t}$:

$$\phi_t=\frac{\beta(1-\gamma)}{2}\frac{\bar{\delta}_t}{1-\bar{\delta}_t}.$$

From the Lagrangian, it follows that the problem can be written recursively with five state variables $(b_{t-1}^*,\tilde{q}_t,n_t^*,\bar{\delta}_{t-1},\xi_{t-1})$, where ξ_t is the Lagrange multiplier on $z_t^e=\mathbb{E}_t z_{t+1}$.

Take the optimality conditions with respect to $\bar{\sigma}_t^2$ and z_t^e:

$$\phi_t=\gamma\mu_t\bar{\omega}(n_t^*+f_t^*-b_t^*)\qquad\text{and}\qquad\xi_t=\beta(1-\gamma)\bar{\delta}_t^2z_t^e+2\phi_t(1-\bar{\delta}_t)^2z_t^e+\gamma\mu_t.$$

Thus, if $\mu_{t-1}=0$, then $\phi_{t-1}=0$, which implies $\delta_{t-1}=\bar{\delta}_{t-1}=0$ and $\xi_{t-1}=0$. It follows that both promise-keeping constraints are not binding (as Lagrange multipliers $\phi_{t-1} = \xi_{t-1} = 0$) and — conditional on the value of b_t^* — the optimal policy does not depend on the previous history of shocks.

If additionally $\mu_t = 0$, then by the same logic $\delta_t = 0$, which according to Proposition 3, coincides with the optimal discretionary policy. Furthermore, the time consistency of FXI from Proposition 1 carries over to the case when interventions might be constrained in the future.

Finally, when $\mu_{t-1} = \mu_t = \mathbb{E}_t\mu_{t+1} = 0$, Theorem 2 implies that the optimal FXI closes the UIP wedge $\mathbb{E}_t\Delta z_{t+1} = 0$, while Theorem 1 ensures that $\delta_t = 0$. ∎

Example with a full analytical solution Consider the case where FX is constrained only at $t = s > 0$ and unconstrained for all other $t \neq s$. Then $\mu_t = 0$ for all $t \neq s$ and $\mu_s \neq 0$. Also denote $\hat{n}_s^* \equiv n_s^* + f_s^*$, which is given exogenously. For simplicity we assume that $\mathbb{E}_t\hat{n}_s^* = \bar{n}_s^*$ for all $t < s$ and, in general, $\bar{n}_s^* \neq \hat{n}_s^*$. We also assume that $\mathbb{E}_t\bar{\sigma}_s^2 = \bar{\sigma}_s^2$ for all $t \leq s$ to avoid extra notation. The system of constraints and optimality conditions that determines the equilibrium path of the economy $\{x_t, f_t^*, z_t, e_t, b_t^*\}$ consists of (11)–(13), (16) and (18). Given this system and $\mu_t = 0$ for $t \neq s$, we have:

$$
\begin{aligned}
x_t &= 0 \quad \forall\, t \neq s+1, \\
x_{s+1} &= -\frac{2\gamma\bar{\omega}}{1-\gamma}\frac{\bar{\omega}\bar{\sigma}_s^2}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}(\hat{n}_s^* - b_s^*)^2(e_{s+1} - \mathbb{E}_s e_{s+1}), \\
\mathbb{E}_t\Delta z_{t+1} &= 0 \quad \forall\, t < s-1 \text{ and } t > s+1, \\
\mathbb{E}_{s-1}\Delta z_s &= -\beta\mathbb{E}_{s-1}\mu_s = -\beta\bar{\omega}\bar{\sigma}_s^2(\bar{n}_s^* - b_s^*), \\
\mathbb{E}_s\Delta z_{s+1} &= (1+\beta+\bar{\omega}\bar{\sigma}_s^2)\mu_s = \bar{\omega}\bar{\sigma}_s^2(\hat{n}_s^* - b_s^*), \\
\mathbb{E}_{s+1}\Delta z_{s+2} &= -\mu_s\big[1 + 2\bar{\omega}(\hat{n}_s^* - b_s^*)(e_{s+1} - \mathbb{E}_s e_{s+1})\big] = -\frac{\bar{\omega}\bar{\sigma}_s^2(\hat{n}_s^* - b_s^*)}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}\big[1 + 2\bar{\omega}(\hat{n}_s^* - b_s^*)(e_{s+1} - \mathbb{E}_s e_{s+1})\big],
\end{aligned}
$$

where we used risk-sharing at $t = s$, $\mathbb{E}_s\Delta z_{s+1} = \bar{\omega}\bar{\sigma}_s^2(\hat{n}_s^* - b_s^*)$, to solve out $\mu_s = \frac{\bar{\omega}\bar{\sigma}_s^2}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}(\hat{n}_s^* - b_s^*)$, and the fact that b_s^* is predetermined at $s-1$. Note that risk-sharing conditions for $t \neq s$ act as side-equations to pin down the value of unconstrained FXI f_t^* that ensures $\mu_t = 0$.

To solve for $\{z_t\}$, it remains to apply the intertemporal budget constraint as of $t \leq s-1$ (prior to realization of uncertainty), at $t = s$ (conditional on realization of $\hat{n}_s^*$), and for $t \geq s+1$ (with full information about $\hat{n}_s^*$ and e_{s+1}), respectively. It can be shown that:

$$
\begin{aligned}
z_t &= -\beta^{s+1}\frac{\bar{\omega}\bar{\sigma}_s^2}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}\bar{\omega}\bar{\sigma}_s^2(\bar{n}_s^* - b_s^*) \quad \forall\, t \leq s-1, \\
z_s &= -\frac{\beta\bar{\omega}\bar{\sigma}_s^2}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}\Big[\big(1+\beta^s\bar{\omega}\bar{\sigma}_s^2\big)(\bar{n}_s^* - b_s^*) - \big(1+\bar{\omega}\bar{\sigma}_s^2\big)(\hat{n}_s^* - \bar{n}_s^*)\Big], \\
z_{s+1} &= \frac{\bar{\omega}\bar{\sigma}_s^2}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}\Big[\big(1+(1-\beta^{s+1})\bar{\omega}\bar{\sigma}_s^2\big)(\bar{n}_s^* - b_s^*) + \big(1+(1-\beta)\bar{\omega}\bar{\sigma}_s^2\big)(\hat{n}_s^* - \bar{n}_s^*) + 2\bar{\omega}\beta(\hat{n}_s^* - b_s^*)^2\tilde{e}_{s+1}\Big], \\
z_t &= \frac{\bar{\omega}\bar{\sigma}_s^2}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}\Big[(1-\beta^{s+1})\bar{\omega}\bar{\sigma}_s^2(\bar{n}_s^* - b_s^*) + (1-\beta)\bar{\omega}\bar{\sigma}_s^2(\hat{n}_s^* - \bar{n}_s^*) - (1-\beta)2\bar{\omega}(\hat{n}_s^* - b_s^*)^2\tilde{e}_{s+1}\Big] \quad \forall\, t \geq s+2,
\end{aligned}
$$

where $\tilde{e}_{s+1} \equiv e_{s+1} - \mathbb{E}_s e_{s+1}$, and:

$$b_s^* = \frac{1-\beta^s}{1-\beta}\frac{\beta\bar{\omega}\bar{\sigma}_s^2}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}\bar{\omega}\bar{\sigma}_s^2(\bar{n}_s^* - b_s^*),$$

which allows us to solve out all endogenous variables and fully characterize the equilibrium $\{z_t, b_t^*\}$. Finally, the surprise exchange rate movement at $t = s+1$ is $\tilde{e}_{s+1} = \tilde{q}_{s+1} - \mathbb{E}_s\tilde{q}_{s+1} + x_{s+1} - (z_{s+1} - \mathbb{E}_s z_{s+1})$, and using the solution for x_{s+1} and z_{s+1} this results in:

$$e_{s+1} - \mathbb{E}_s e_{s+1} = \frac{\tilde{q}_{s+1} - \mathbb{E}_s\tilde{q}_{s+1}}{1 + 2\bar{\omega}\frac{\gamma+\beta(1-\gamma)}{1-\gamma}\frac{\bar{\omega}\bar{\sigma}_s^2}{1+\beta+\bar{\omega}\bar{\sigma}_s^2}(\hat{n}_s^* - b_s^*)^2},$$

which allows to evaluate σ_s^2. We plot this solution for given values of $\bar{n}_s^*$ and $\hat{n}_s^*$ in Appendix Figure A2.

A4 Derivations and Proofs for Sections 4

A4.1 Taxes and international transfers (Section 4.1)

Taxes and returns There are three types of agents and their portfolio returns are given by:

1. Domestic households save in home-currency bonds B_t and earn a return $R_t/(1+\tau_t^h)$. Their Euler equation is given by:

$$\frac{R_t}{1+\tau_t^h}\mathbb{E}_t\left\{\Theta_{t+1}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right\} = 1, \qquad \Theta_{t+1} = \beta\frac{C_{Tt}}{C_{Tt+1}}.$$

2. Domestic financial agents — noise traders and intermediaries, respectively — invest $(1+\tau_{Ht}^*)N_{Ht}^*/R_t^*$ and $(1+\tau_{Ht}^*)D_{Ht}^*/R_t^*$ dollars in a carry trade position with a return $\tilde{R}_{Ht+1}^* = \frac{R_t^*}{1+\tau_{Ht}^*} - \frac{R_t}{1+\tau_{Ht}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$. While N_{Ht}^* is exogenous, intermediaries' portfolio choice solves an optimization problem that gives rise to (4), and thus in the presence of taxes satisfies:

$$\frac{D_{Ht}^*}{R_t^*} = \frac{\mathbb{E}_t\Theta_{t+1}\tilde{R}_{Ht+1}^*}{(1+\tau_{Ht}^*)\omega_H\sigma_{Ht}^2}, \qquad \sigma_{Ht}^2 = \left(\frac{R_t}{1+\tau_{Ht}}\right)^2\mathrm{var}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = \frac{\sigma_t^2}{(1+\tau_{Ht})^2}.$$

Note that $(1+\tau_{Ht})D_{Ht}/R_t = -(1+\tau_{Ht}^*)\mathcal{E}_t D_{Ht}^*/R_t^*$ is the home-currency position of their zero-capital portfolio, where D_{Ht}^* and D_{Ht} are units of the zero-coupon bonds purchased in each currency, and similarly for the noise traders.

3. Foreign financial agents invest N_{Ft}^*/R_t^* and D_{Ft}^*/R_t^*, respectively, in a carry trade with a return $\tilde{R}_{Ft+1}^* = R_t^* - \frac{R_t}{1+\tau_{Ft}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$. While N_{Ft}^* is exogenous, intermediaries' portfolio choice satisfies:

$$\frac{D_{Ft}^*}{R_t^*} = \frac{\mathbb{E}_t\Theta_{t+1}\tilde{R}_{Ft+1}^*}{\omega_F\sigma_{Ft}^2}, \qquad \sigma_{Ft}^2 = \left(\frac{R_t}{1+\tau_{Ft}}\right)^2\mathrm{var}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = \frac{\sigma_t^2}{(1+\tau_{Ft})^2}.$$

Their home-currency position is given by $(1+\tau_{Ft})D_{Ft}/R_t = -\mathcal{E}_t D_{Ft}^*/R_t^*$, and similarly for the noise traders.

We further assume that ω is a common risk aversion parameter and m_H and m_F are masses of home and foreign arbitrageurs such that $m_H + m_F = 1$. Therefore, $\omega_H = \omega/m_H$ and $\omega_F = \omega/m_F$

represent the inverse aggregate risk absorption capacities by home and foreign intermediaries, repstectively, where $\frac{1}{m_H}\frac{D^*_{Ht}}{R^*_t}$ and $\frac{1}{m_F}\frac{D^*_{Ft}}{R^*_t}$ correspond to individual positions. Combining the household Euler equation with the two portfolio choice conditions:

$$\frac{\beta R^*_t}{1+\tau^*_{Ht}}\mathbb{E}_t\frac{C_{Tt}}{C_{Tt+1}} = \frac{1+\tau^h_t}{1+\tau_{Ht}} + \frac{\omega\sigma^2_t}{m_H}\frac{1+\tau^*_{Ht}}{(1+\tau_{Ht})^2}\frac{D^*_{Ht}}{R^*_t},$$

$$\beta R^*_t\mathbb{E}_t\frac{C_{Tt}}{C_{Tt+1}} = \frac{1+\tau^h_t}{1+\tau_{Ft}} + \frac{\omega\sigma^2_t}{m_F}\frac{1}{(1+\tau_{Ft})^2}\frac{D^*_{Ft}}{R^*_t}.$$

Express out $\omega\sigma^2_t\frac{D^*_{Ht}}{R^*_t}$ and $\omega\sigma^2_t\frac{D^*_{Ft}}{R^*_t}$ and add the two resulting equations to solve out $D^*_{Ht}+D^*_{Ft}$ using market clearing $D^*_{Ft}+D^*_{Ht} = B^*_t - F^*_t - N^*_{Ft} - N^*_{Ht}$ to obtain:

$$\beta R^*_t\mathbb{E}_t\frac{C_{Tt}}{C_{Tt+1}} = (1+\tau^h_t)\frac{m_H\frac{1+\tau_{Ht}}{1+\tau^*_{Ht}} + m_F(1+\tau_{Ft})}{m_H\left(\frac{1+\tau_{Ht}}{1+\tau^*_{Ht}}\right)^2 + m_F(1+\tau_{Ft})^2} + \frac{\omega\sigma^2_t}{m_H\left(\frac{1+\tau_{Ht}}{1+\tau^*_{Ht}}\right)^2 + m_F(1+\tau_{Ft})^2}\frac{B^*_t - F^*_t - N^*_t}{R^*_t}.$$

Assuming either (i) $\tau_{Ht} = \tau_{Ft} = \tau_t$ and $\tau^*_{Ht} = 0$ or (ii) $\tau_{Ft} = \tau_t$, $\tau^*_{Ht} = \frac{-\tau_t}{1-\tau_t}$ and $\tau_{Ht} = 0$ results in (19) in the text.

Taxes and the country budget constraint The net revenues of the government combine returns on FXI and taxes imposed on arbitrageurs and noise traders:

$$\begin{aligned} T^g_t &= \left(F_{t-1} - \frac{F_t}{R_t}\right) + \mathcal{E}_t\left(F^*_{t-1} - \frac{F^*_t}{R^*_t}\right) + \tau^h_t\frac{B_t}{R_t} + \tau_{Ht}\frac{D_{Ht}+N_{Ht}}{R_t} + \tau_{Ft}\frac{D_{Ft}+N_{Ft}}{R_t} + \tau^*_{Ht}\mathcal{E}_t\frac{D^*_{Ht}+N^*_{Ht}}{R^*_t} \\ &= \left[R^*_{t-1} - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right]\mathcal{E}_t\frac{F^*_{t-1}}{R^*_{t-1}} + \tau^h_t\frac{B_t}{R_t} + \frac{\tau^*_{Ht}-\tau_{Ht}}{1+\tau_{Ht}}\mathcal{E}_t\frac{D^*_{Ht}+N^*_{Ht}}{R^*_t} - \frac{\tau_{Ft}}{1+\tau_{Ft}}\mathcal{E}_t\frac{D^*_{Ft}+N^*_{Ft}}{R^*_t}, \end{aligned}$$

where we used the zero net position of arbitrageurs, noise traders and the central bank. The budget constraint of the households after netting out the expenditure on non-tradables is given by:

$$(1+\tau^h_t)\frac{B_t}{R_t} - B_{t-1} = \mathcal{E}_t(Y_{Tt} - C_{Tt}) + T^g_t + T^f_t,$$

where the profits of the local financial sector are given by

$$T^f_t = (D_{Ht-1}+N_{Ht-1}) + \mathcal{E}_t(D^*_{Ht-1}+N^*_{Ht-1}) = \left[R^*_{t-1} - \frac{1+\tau^*_{Ht-1}}{1+\tau_{Ht-1}}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}R_{t-1}\right]\mathcal{E}_t\frac{D^*_{Ht-1}+N^*_{Ht-1}}{R^*_{t-1}}.$$

Combining these three equations together, we obtain the country budget constraint:

$$\begin{aligned} \frac{B_t}{R_t} =& B_{t-1} + \mathcal{E}_t(Y_{Tt} - C_{Tt}) + \left[R^*_{t-1} - \frac{1+\tau^*_{Ht-1}}{1+\tau_{Ht-1}}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}R_{t-1}\right]\mathcal{E}_t\frac{D^*_{Ht-1}+N^*_{Ht-1}}{R^*_{t-1}} \\ &+ \left[R^*_{t-1} - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right]\mathcal{E}_t\frac{F^*_{t-1}}{R^*_{t-1}} + \frac{\tau^*_{Ht}-\tau_{Ht}}{1+\tau_{Ht}}\mathcal{E}_t\frac{D^*_{Ht}+N^*_{Ht}}{R^*_t} - \frac{\tau_{Ft}}{1+\tau_{Ft}}\mathcal{E}_t\frac{D^*_{Ft}+N^*_{Ft}}{R^*_t}, \end{aligned}$$

The market clearing condition for home bonds, $B_t + D_{Ht} + D_{Ft} + N_{Ht} + N_{Ft} + F_t = 0$, can be rewritten using the zero net positions of all financial traders as:

$$\frac{B_t}{R_t} = \frac{1+\tau_{Ht}^*}{1+\tau_{Ht}}\mathcal{E}_t\frac{D_{Ht}^*+N_{Ht}^*}{R_t^*} + \frac{1}{1+\tau_{Ft}}\mathcal{E}_t\frac{D_{Ft}^*+N_{Ft}^*}{R_t^*} + \mathcal{E}_t\frac{F_t^*}{R_t^*}.$$

Using this expression, substitute B_t and B_{t-1} out of the country's budget constraint and simplify:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt} - \left[R_{t-1}^* - \frac{R_{t-1}}{1+\tau_{Ft-1}}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right]\frac{D_{Ft-1}^*+N_{Ft-1}^*}{R_{t-1}^*},$$

where we used the definition of the NFA position of the country, $B_{t-1}^* = F_t^*+D_{Ft}^*+D_{Ht}^*+N_{Ft}^*+N_{Ht}^*$. The last term in this budget constraint corresponds to the international transfer of income and can be rewritten using the definition of $\tilde{R}_{Ft+1}^*$ and the optimal portfolio choice of foreign arbitrageurs:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt} - \tilde{R}_{Ft}^*\left(\frac{m_F\mathbb{E}_{t-1}[\Theta_t\tilde{R}_{Ft}^*]}{\omega\sigma_{t-1}^2/(1+\tau_{Ft-1})^2} + \frac{N_{Ft-1}^*}{R_{t-1}^*}\right).$$

For the rest of the analysis, we assume that $\tau_{Ft} = \frac{-\tau_{Ht}^*}{1+\tau_{Ht}^*} = \tau_t$ and $\tau_t^h = \tau_{Ht} = 0$ so that the carry-trade returns are given by $\tilde{R}_{t+1}^* = \tilde{R}_{Ft+1}^* = R_t^* - \frac{R_t}{1+\tau_t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$.

Lemma A5 *The second order Taylor expansion around a zero-NFA steady state of the welfare loss for any budget- and resource-feasible allocation relative to the allocation $\{\tilde{C}_{Nt},\tilde{C}_{Tt}\}$ with $\tilde{R}_t^*=0$ is given by:*

$$\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\gamma z_t^2 + (1-\gamma)x_t^2 + 2\beta\gamma\left(\frac{m_F}{\bar{\omega}\bar{\sigma}_t^2}\psi_t - n_{Ft}^*\right)\psi_t\right],$$

where $\psi_t = i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} - \tau_t$. Therefore, it is sufficient to know the first-order dynamics $\{x_t, z_t, \psi_t, \bar{\omega}\bar{\sigma}_t^2\}$ to evaluate the second-order welfare loss.

Proof: We use the same notation as in the proof of Lemma A1. The second-order approximation for the non-tradable sector is the same as before and, therefore, we focus exclusively on the tradable sector. Rewrite the country's budget constraint in terms of deviations as

$$\beta e^{-r_t^*}\check{b}_t^* - \check{b}_{t-1}^* = e^{y_{Tt}} - e^{c_{Tt}} - \tilde{R}_t^*\left(\frac{m_F\beta\mathbb{E}_{t-1}[e^{-\Delta c_{Tt}}\tilde{R}_t^*]}{\bar{Y}_T\omega\sigma_{t-1}^2/(1+\tau_{t-1})^2} + \beta e^{-r_{t-1}^*}n_{Ft-1}^*\right),$$

where we used $\check{b}_t^* = B_t^*/\bar{Y}_T$, $n_t^* = N_t^*/\bar{Y}_T$, $\bar{C}_T = \bar{Y}_T$, $\bar{\Theta}=\beta$, $\bar{R}^* = 1/\beta$, $\bar{\tilde{R}}^*=0$, $\bar{N}^*=0$, $\bar{\tau}=0$. The second-order Taylor expansion to the flow budget constraint is:

$$0 = \check{b}_{t-1}^* + y_{Tt} + \frac{1}{2}y_{Tt}^2 - c_{Tt} - \frac{1}{2}c_{Tt}^2 - \beta\check{b}_t^* + \beta r_t^*\check{b}_t^* - \tilde{r}_t^*\frac{m_F\mathbb{E}_{t-1}\tilde{r}_t^*}{\bar{\omega}\bar{\sigma}_{t-1}^2} - \tilde{r}_t^*n_{Ft-1}^* + h.o.t.,$$

where we denoted $\tilde{r}_t^* \equiv \beta\tilde{R}_t^*$ and used the result from Lemma A3 that $\omega\sigma_t^2 = \frac{1}{\beta\bar{Y}_T}\bar{\omega}\bar{\sigma}_t^2 + \mathcal{O}(\nu)$.[55]

[55] The fact that Lemma A3 still applies in this extended environment follows from the same first-order Taylor expansion of the equilibrium system for the goods market clearing (2), the risk-sharing condition (19), and the country budget constraint which is still $\beta\check{b}_t^* = \check{b}_{t-1}^* + y_{Tt} - c_{Tt} + \mathcal{O}(\nu)$ up to first order. That is, international income transfers are of the second order and only affect the second-order approximation to the welfare, but not the first-order equilibrium dynamic system.

Integrate the flow budget constraint across periods and impose the transversality condition to get

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_{Tt} = \check{b}^*_{-1} + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[y_{Tt} + \frac{1}{2} y_{Tt}^2 - \frac{1}{2} c_{Tt}^2 + \beta r_t^* \check{b}_t^* - \tilde{r}_t^* \frac{m_F \mathbb{E}_{t-1} \tilde{r}_t^*}{\bar{\omega} \bar{\sigma}_{t-1}^2} - \tilde{r}_t^* n^*_{Ft-1} \right] + h.o.t.$$

Recall that the optimal allocation $\{\tilde{c}_{Tt}, \tilde{b}_t^*, \bar{\omega}\tilde{\sigma}_t^2\}$ with $\tilde{r}_t^* = 0$ also satisfies the intertemporal budget constraint:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \tilde{c}_{Tt} = \tilde{b}^*_{-1} + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[y_{Tt} + \frac{1}{2} y_{Tt}^2 - \frac{1}{2} \tilde{c}_{Tt}^2 + \beta r_t^* \tilde{b}_t^* \right] + h.o.t.$$

Take the difference between the two equations to obtain

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_{Tt} - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \tilde{c}_{Tt} = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[z_t^2 + 2\tilde{r}_t^* \left(\frac{m_F \mathbb{E}_{t-1} \tilde{r}_t^*}{\bar{\omega} \bar{\sigma}_{t-1}^2} + n^*_{Ft-1} \right) \right] + h.o.t.,$$

where we define $b_t^* \equiv \check{b}_t^* - \tilde{b}_t^*$ and we use the result from the proof of Lemma A1 that

$$\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[(c_{Tt} - \tilde{c}_{Tt}) \tilde{c}_{Tt} - \beta r_t^* b_t^* \right] = 0 + h.o.t.,$$

which we use to obtain $\frac{1}{2} z_t^2 = \frac{1}{2}(c_{Tt} - \tilde{c}_{Tt})^2$ from $\frac{1}{2} c_{Tt}^2 - \frac{1}{2} \tilde{c}_{Tt}^2 = \frac{1}{2} z_t^2 + (c_{Tt} - \tilde{c}_{Tt}) \tilde{c}_{Tt}$.

Following the same remaining steps as in the proof of Lemma A1, the second-order approximation to the welfare loss relative to the first-best allocation is:

$$\tilde{\mathbb{W}}_0 - \mathbb{W}_0 = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma z_t^2 + (1-\gamma) x_t^2 + 2\gamma (\mathbb{E}_{t-1} \tilde{r}_t^*) \left(\frac{m_F \mathbb{E}_{t-1} \tilde{r}_t^*}{\beta \bar{\omega} \bar{\sigma}_{t-1}^2} + n^*_{Ft-1} \right) \right] + h.o.t.$$

where we rewrote $\mathbb{E}_0 \tilde{r}_t^* = \mathbb{E}_0 \mathbb{E}_{t-1} \tilde{r}_t^*$ by the law of iterated expectation. Finally, note that:

$$\mathbb{E}_{t-1} \tilde{r}_t^* = \mathbb{E}_{t-1} \left[\beta R^*_{t-1} - \frac{\beta R_{t-1}}{1 + \tau_{t-1}} \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t} \right] = i^*_{t-1} - i_{t-1} + \mathbb{E}_{t-1} \Delta e_t + \tau_{t-1} + h.o.t. = -\psi_{t-1} + h.o.t.$$

and rewrite the loss function as (since by construction $\psi_{-1} = 0$):

$$\tilde{\mathbb{W}}_0 - \mathbb{W}_0 = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma z_t^2 + (1-\gamma) x_t^2 + 2\beta\gamma \left(\frac{m_F}{\bar{\omega} \bar{\sigma}_t^2} \psi_t - n^*_{Ft} \right) \psi_t \right] + h.o.t. \qquad \blacksquare$$

Proof of Proposition 6 The proof of Lemma A5 shows that international transfers are of second order and therefore, to the first-order approximation, the country budget constraint remains the same as in the baseline model. The goods market clearing condition is exactly the same as before and the risk-sharing condition (19) now includes capital controls τ_t, and its first-order Taylor expansion in terms of deviations from the first best $\{\tilde{C}_{Tt}, \tilde{B}_t^*\}$, around a steady state with $\bar{\tau} = 0$, is:

$$\mathbb{E}_t \Delta z_{t+1} = \tau_t + \bar{\omega} \bar{\sigma}_t^2 (n_t^* + f_t^* - b_t^*),$$

following the same steps as in the proof of Lemma A3. Using the definition of the expected carry trade return, $\psi_t \equiv i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} - \tau_t$, and the household Euler equation:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta z_{t+1} = \psi_t + \tau_t,$$

which corresponds to equation (20) in the text and implies $\psi_t = \bar{\omega}\bar{\sigma}_t^2(n_t^* + f_t^* - b_t^*)$.[56]

The planner's problem combines the second-order loss function and the first-order constraints:

$$\begin{aligned}
\min_{\{x_t, f_t^*, z_t, e_t, b_t^*, \bar{\sigma}_t^2, \psi_t, \tau_t\}} \quad & \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma z_t^2 + (1-\gamma) x_t^2 + 2\beta\gamma \left(\frac{m_F}{\bar{\omega}\bar{\sigma}_t^2} \psi_t - n_{Ft}^* \right) \psi_t \right] \\
\text{subject to} \qquad \beta b_t^* &= b_{t-1}^* - z_t, \\
e_t &= \tilde{q}_t + x_t - z_t, \\
\mathbb{E}_t \Delta z_{t+1} &= \tau_t + \psi_t, \\
\psi_t &= \bar{\omega}\bar{\sigma}_t^2 (n_t^* + f_t^* - b_t^*) \quad \text{with} \quad \bar{\sigma}_t^2 = \mathrm{var}_t(e_{t+1}),
\end{aligned}$$

where $n_t^* = n_{Ht}^* + n_{Ft}^*$.

Part (a): From the planner's problem, note that the undistorted allocation without international transfers (as in Proposition 1):

$$x_t = z_t = b_t^* = \psi_t = 0 \quad \text{and} \quad e_t = \tilde{q}_t$$

is still feasible, and must be implemented with FXI $f_t^* = -n_t^*$ and without the use of capital controls $\tau_t = 0$. Furthermore, this is the best allocation if $n_{Ft}^* = 0$ independently of the value of $m_F \in [0, 1]$, that is when noise traders are entirely domestic, as in this case the international transfer is always away from the country in expectation and is proportional to ψ_t^2.

Part (b): When $n_{Ft}^* \neq 0$, the planner can improve over this outcome. For example, the planner can still achieve the undistorted macroeconomic allocation, $x_t = z_t = b_t^* = 0$, yet generate a positive expected transfer from the rest of the world, $\big(\frac{m_F}{\bar{\omega}\bar{\sigma}_t^2}\psi_t - n_{Ft}^*\big)\psi_t < 0$. When $x_t = z_t = 0$, then $e_t = \tilde{q}_t$ and hence $\bar{\sigma}_t^2 = \mathrm{var}_t(\tilde{q}_{t+1})$ is given exogenously. In this case, the maximum transfer from abroad equals $\frac{\bar{\omega}\bar{\sigma}_t^2}{4m_F}(n_{Ft}^*)^2 > 0$, and it is achieved when the carry trade return equals $\psi_t = \frac{\bar{\omega}\bar{\sigma}_t^2}{2m_F} n_{Ft}^*$. The implementation of this outcome requires both the use of FXI and capital controls. In particular, $\tau_t = -\psi_t$ is required to ensure no risk-sharing distortion, $z_t = 0$, and FXI underreact to $n_{Ft}^* \neq 0$ shocks to ensure $\psi_t \neq 0$. Specifically, the optimal FXI are $f_t^* = -n_{Ht}^* - \big(1 - \frac{1}{2m_F}\big)n_{Ft}^*$. When all intermediaries are foreign, $m_F = 1$, this simplifies to $f_t^* = -n_{Ht}^* - \frac{1}{2}n_{Ft}^*$.

Part (c): If the planner is willing to compromise $x_t = z_t = 0$, i.e. allow for macroeconomic distortions, she can improve the allocation even further by inducing larger transfers from abroad at the cost

[56] In the absence of household savings taxes, $\tau_t^h = 0$, the household Euler equation for domestic bond can be written as $\beta R_t \mathbb{E}_t\big\{\frac{C_{Tt}}{C_{Tt+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\big\} = 1$, and the first best risk-sharing satisfies $\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1$, which together imply $\mathbb{E}_t \Delta z_{t+1} = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$, which is the UIP deviation from the perspective of the household. In contrast, ψ_t is the carry trade return from the perspective of the financial sector, whether home or foreign (provided that $1 + \tau_{Ft} = 1/(1+\tau_{Ht}^*)$ and $\tau_{Ht} = 0$). Under these circumstances, $\psi_t/(\bar{\omega}\bar{\sigma}_t^2)$ is the currency position taken by every intermediary (home or foreign), and there is a unit continuum of them ($m_H + m_F = 1$). Therefore, $\psi_t/(\bar{\omega}\bar{\sigma}_t^2) = n_t^* + f_t^* - b_t^*$ ensures currency market clearing.

of some production and risk-sharing distortions. For simplicity, we consider here the case when the financial sector is entirely foreign, that is $m_F = 1$ and $n_t^* = n_{Ft}^*$. Note that the risk-sharing condition is a side equation that pins down τ_t and the equation for ψ_t determines f_t^*, so that these two constraints can be eliminated from the planner's problem. Furthermore, ψ_t only appears in the objective function, and the first-order optimality for ψ_t requires $\psi_t = \frac{1}{2}\bar{\omega}\bar{\sigma}_t^2 n_{Ft}^*$.

Substituting the optimal ψ_t back into the loss function, we can rewrite the planner's problem as:

$$\min_{\{x_t, z_t, b_t^*\}} \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma z_t^2 + (1-\gamma)x_t^2 + \frac{\beta\gamma}{2}(n_{Ft}^*)^2\bar{\omega}\bar{\sigma}_t^2\right]$$

subject to $\beta b_t^* = b_{t-1}^* - z_t$ and with $\bar{\sigma}_t^2 \equiv \mathbb{E}_t\big(\tilde{q}_{t+1} + x_{t+1} - z_{t+1}\big)^2 - \big[\mathbb{E}_t(\tilde{q}_{t+1} + x_{t+1} - z_{t+1})\big]^2$. Let λ_t denote the Lagrange multiplier on the budget constraint and take the first order conditions with respect to b_t^*, z_t and x_t:

$$\begin{aligned}
\lambda_t &= \mathbb{E}_t\lambda_{t+1}, \\
\lambda_t &= 2\gamma z_t + \gamma(n_{Ft-1}^*)^2\bar{\omega}\big[\tilde{q}_t + x_t - z_t - \mathbb{E}_{t-1}\big(\tilde{q}_t + x_t - z_t\big)\big], \\
0 &= 2(1-\gamma)x_t - \gamma(n_{Ft-1}^*)^2\bar{\omega}\big[\tilde{q}_t + x_t - z_t - \mathbb{E}_{t-1}\big(\tilde{q}_t + x_t - z_t\big)\big].
\end{aligned}$$

The latter condition can be rewritten as

$$x_t = \frac{\gamma\bar{\omega}}{2(1-\gamma)}(n_{Ft-1}^*)^2(e_t - \mathbb{E}_{t-1}e_t).$$

This implies that the planner does not distort the average output gap, $\mathbb{E}_{t-1}x_t = 0$, but in states of the world with relatively depreciated exchange rate $e_t > \mathbb{E}_{t-1}e_t$, the monetary policy overstimulates the economy $x_t > 0$, which further weakens the currency and increases the volatility of the exchange rate $\bar{\sigma}_t^2$.

Further, note that

$$\mathbb{E}_t\lambda_{t+1} = 2\gamma\mathbb{E}_t z_{t+1} + \gamma\bar{\omega}(n_{Ft}^*)^2\mathbb{E}_t(e_{t+1} - \mathbb{E}_t e_{t+1}) = 2\gamma\mathbb{E}_t z_{t+1},$$

and, therefore, the first two optimality conditions imply

$$\mathbb{E}_t\Delta z_{t+1} = \frac{\bar{\omega}}{2}(n_{Ft-1}^*)^2(e_t - \mathbb{E}_{t-1}e_t).$$

It follows that the planner distorts international risk sharing: in states of the world with a relatively depreciated exchange rate, $e_t > \mathbb{E}_{t-1}e_t$, it is optimal to introduce a UIP wedge, $\mathbb{E}_t\Delta z_{t+1} > 0$, so that z_t falls and the exchange rate depreciates even further leading to a higher volatility $\bar{\sigma}_t^2$. Recall that given the path of ψ_t implemented with FXI, the desired deviations from risk sharing, $\mathbb{E}_t\Delta z_{t+1} = \tau_t + \psi_t \neq 0$, are ensured using capital controls τ_t.

These policy interventions lower the elasticity of currency supply by foreign intermediaries that are faced with a greater equilibrium exchange rate volatility $\bar{\sigma}_t^2$. This allows the planner to extract greater rents from foreign noise-trade currency demand n_{Ft}^*. ■

A4.2 Multi-country model (Section 4.2)

Consider a world of a unit continuum of small open economies $i \in [0,1]$, each as described in Section 2.1. Given exogenous shocks as well as monetary and FX policies in each economy, the global equilibrium is determined by household optimality conditions (2) and (3):

$$\frac{\gamma}{1-\gamma}\frac{C_{Nit}}{C_{Tit}} = \frac{\mathcal{E}_{it}P^*_{Tt}}{P_{Nit}} \qquad \text{and} \qquad \beta R_{it}\mathbb{E}_{it}\frac{C_{Nit}}{C_{Nit+1}} = 1,$$

the international risk-sharing conditions (7):

$$\beta R^*_t\mathbb{E}_t\left\{\frac{C_{Tit}}{C_{Tit+1}}\frac{P^*_{Tt}}{P^*_{Tt+1}}\right\} = 1 + \omega_i\sigma^2_{it}\frac{B^*_{it}-N^*_{it}-F^*_{it}}{R^*_t} \qquad \text{with} \quad \sigma^2_{it} = R^2_{it}\cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_{it}}{\mathcal{E}_{it+1}}\Big),$$

and the countries' budget constraints (6):

$$\frac{B^*_{it}}{R^*_t} - B^*_{it-1} = P^*_{Tt}(Y_{Tit} - C_{Tit})$$

for all i and t, as well as the new conditions for the global market clearing for tradables and bonds for every t:

$$\int_0^1 C_{Tit}\mathrm{d}i = \int_0^1 Y_{Tit}\mathrm{d}i \equiv Y_{Tt} \qquad \text{and} \qquad \int_0^1 B^*_{it}\mathrm{d}i = 0.$$

Currencies $i \in [0, m_0]$ peg to the dollar so that $\mathcal{E}_{it} = 1$.

The problem of a global planner that takes as given the structure of international asset markets is:[57]

$$\max_{\{\{C_{Nit},C_{Tit},B^*_{it}\},R^*_t,P^*_{Tt+1}\}} \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_0^1\left[\gamma\log C_{Tit} + (1-\gamma)\left(\log C_{Nit} - \frac{C_{Nit}}{A_{it}}\right)\right]\mathrm{d}i$$

$$\text{subject to} \qquad \frac{B^*_{it}}{R^*_t} - B^*_{it-1} = P^*_{Tt}(Y_{Tit} - C_{Tit}),$$

$$\int_0^1 B^*_{it}\mathrm{d}i = 0.$$

We denote the globally efficient allocation with hat's and contrast it with the non-cooperative efficient allocation denoted with tilde's.

Lemma A6 *Up to the first order approximation, the local first-best allocation $\{\tilde{c}_{Nit}, \tilde{c}_{Tit}, \tilde{b}_{it}\}$ from Section 2.2 coincides with the global efficient allocation $\{\hat{c}_{Nit}, \hat{c}_{Tit}, \hat{b}_{it}\}$.*

Proof: Taking the optimality conditions in the planner's problem, we get that the optimal output in the non-tradable sector is the same with and without cooperation, $\hat{C}_{Nit} = \tilde{C}_{Nit} = A_{it}$. The optimality

[57] Note that the global planner is allowed to manipulate equilibrium prices R^*_t and P^*_{Tt}, but is not allowed to change budget sets of countries beyond the pecuniary effect of market prices that are common for all countries (see Dávila, Hong, Krusell, and Ríos-Rull 2012, Itskhoki and Moll 2019).

conditions with respect to C_{Tit}, B^*_{it}, P^*_{Tt} and R^*_t are given by

$$\begin{aligned}
\lambda_{it} &= \frac{1}{\hat{P}^*_{Tt}\hat{C}_{Tit}}, \\
\lambda_{it} &= \beta \hat{R}^*_t \mathbb{E}_t \lambda_{it+1} + \hat{R}^*_t \mu_t, \\
0 &= \int_0^1 \lambda_{it}(Y_{Tit} - \hat{C}_{Tit})\mathrm{d}i, \\
0 &= \int_0^1 \lambda_{it}\hat{B}^*_{it}\mathrm{d}i.
\end{aligned}$$

Assuming that countries have zero initial positions $B^*_{i,-1} = 0$, the countries' budget constraints and the market clearing for bonds imply that the latter two optimality conditions are isomorphic and one of them can be dropped as redundant. Consider a steady state with $\bar{B}^*_i = 0$ as a point of approximation. We allow for an arbitrary small cross-country variation in $\bar{Y}_{Ti}$, but then take the limit $\bar{Y}_{Ti} \to \bar{Y}_T$. The last optimality condition is then automatically satisfied. It follows from the budget constraint that $\bar{C}_{Ti} = \bar{Y}_{Ti}$ and hence, the former two optimality conditions require that $1 - \beta\bar{R}^* = \bar{\mu}\bar{R}^*\bar{P}^*_T\bar{Y}_{Ti}$. Given that the right-hand side of the equation varies with i, the unique solution is $\bar{\mu} = 0$ and $\beta\bar{R}^* = 1$.

Taking the first-order approximation around this steady state, we get a linearized Euler equation

$$\mathbb{E}_t \Delta \hat{c}_{it} = \hat{r}^*_t + \frac{\bar{P}_T \bar{Y}_T}{\beta}\mu_t, \tag{A9}$$

where with a slight abuse of the notation, μ_t is the first-order deviation from zero. Also note that from here on we drop the tradable index T on all log deviations, $c_{it} := c_{Tit}$ and $y_{it} := y_{Tit}$. Since the right-hand side of (A9) does not vary with i, we can integrate it across all i and impose the market clearing condition, $\int_0^1 \hat{c}_{it}\mathrm{d}i = y_{Tt}$, resulting in the optimal cooperative risk-sharing condition for all i:

$$\mathbb{E}_t \Delta \hat{c}_{it} = \mathbb{E}_t \Delta y_{Tt+1}.$$

These equations together with the linearized budget constraint for all i:

$$\beta \hat{b}^*_{it} = \hat{b}^*_{it} + y_{it} - \hat{c}_{it},$$

where $\hat{b}^*_{it} \equiv B^*_{it}/\bar{Y}_T$, and the transversality condition, uniquely pin down the globally efficient allocation of tradables $\{\hat{c}_{it}, \hat{b}_{it}\}$.

These conditions coincide with the equilibrium system describing the non-cooperative first best in Section A2.2 where the global real interest rate is equal $r^*_t = \mathbb{E}_t\tilde{c}_{it+1} = \mathbb{E}_t\Delta y_{Tt+1}$. Thus, under this condition, consumption of tradables is the same in the optimal cooperative and non-cooperative allocations, $\hat{c}_{it} = \tilde{c}_{it}$, and this allocation is implemented with a global real interest rate $\hat{r}^*_t = \mathbb{E}_t\Delta y_{Tt+1}$ and $\mu_t = 0$. In other words, no tax wedges are needed to implement the global planner's allocation, at least to a first order. Intuitively, when the point of approximation is $\bar{B}_i = 0$, there are no first-order valuation effects in the budget constraint and, hence, no pecuniary redistribution across economies. The optimal intertemporal consumption smoothing chosen by the global planner is the same as in the non-cooperative equilibrium. ■

Implementation Under sticky prices, $P_{Nit}=1$, implementing the global planner's allocation requires the following path of nominal interest rates, prices and exchange rates: (i) for the US, $R_{0t}=R^*_t$ and $P_{T0t}=P^*_{Tt}$, where:

$$\beta R^*_t \mathbb{E}_t \frac{A_{0t}}{A_{0t+1}} = 1, \qquad P^*_{Tt} = \frac{\gamma}{1-\gamma}\frac{A_{0t}}{\hat{C}_{T0t}},$$

and (ii) for all other countries $i\in(0,1]$:

$$\beta R_{it} \mathbb{E}_t \frac{A_{it}}{A_{it+1}} = 1, \qquad \mathcal{E}_{it} = \frac{\gamma}{1-\gamma}\frac{A_{it}}{P^*_{Tt}\hat{C}_{Tit}}.$$

The ex post and ex ante real interest rates are, respectively, given by $R^*_t\frac{P^*_{Tt}}{P^*_{Tt+1}}$ and $R^*_t\mathbb{E}_t\frac{P^*_{Tt}}{P^*_{Tt+1}}$, and we write the latter in log deviations as $r^*_t = i^*_t - \mathbb{E}_t\pi^*_{Tt+1}$, where $i^*_t \equiv \log(R^*_t/\bar{R}^*) = -\mathbb{E}_t\Delta a_{0t+1}$.

When financial intermediation is also frictional, additionally accommodating currency demand shocks country-by-country, $F^*_{it}=B^*_{it}-N^*_{it}$, still allows to implement the first-best allocation.

Proof of Lemma 1 Define the global risk-sharing wedge $\hat{z}_{it}\equiv c_{it}-\hat{c}_{it}$, where c_{it} is an arbitrary path of tradable consumption that satisfies the feasibility constraints and $\hat{c}_{it}$ is the globally efficient level. As before, we focus on a steady state with $\bar{B}_i=0$ and $\bar{C}_{Ti}=\bar{Y}_{Ti}=\bar{Y}_T$. The first-order approximation to the market clearing condition implies that

$$\int_0^1 c_{it}\mathrm{d}i = y_{Tt} = \int_0^1 \hat{c}_{it}\mathrm{d}i,$$

and, therefore, $\int_0^1 \hat{z}_{it}\mathrm{d}i = 0$. Following the results from Section A2.2, we linearize the risk-sharing condition to get:

$$\mathbb{E}_t\Delta c_{it+1} = r^*_t + \psi_{it}, \qquad \text{where} \quad \psi_{it}\equiv\bar{\omega}_i\bar{\sigma}^2_{it}(n^*_{it}+f^*_{it}-b^*_{it}), \quad \bar{\sigma}^2_{it}\equiv \mathrm{var}_t(e_{it+1}).$$

The proof of Lemma A6 above shows that the efficient allocation satisfies $\mathbb{E}_t\Delta\hat{c}_{it+1}=\mathbb{E}_t\Delta y_{Tt+1}\equiv\hat{r}^*_t$. Subtracting this expression from the previous one allows rewriting the risk-sharing condition in terms of the deviations $\hat{z}_{it}$:

$$\mathbb{E}_t\Delta\hat{z}_{it+1} = r^*_t - \hat{r}^*_t + \psi_{it}.$$

Integrating across i and using the market clearing condition, we get that

$$r^*_t - \hat{r}^*_t = -\bar{\psi}_t \qquad \text{and} \qquad \mathbb{E}_t\Delta\hat{z}_{it+1} = \psi_{it} - \bar{\psi}_t, \qquad \text{where} \quad \bar{\psi}_t \equiv \int_0^1 \psi_{it}\mathrm{d}i.$$

Finally, the second-order approximation to the objective function can be derived using Lemma A2.

To this end, define:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \Bigg\{ \gamma \log C_{Tit} + (1-\gamma)\left(\log C_{Nit} - \frac{C_{Nit}}{A_{it}}\right) + \lambda_{it}\left(P^*_{Tt}(Y_{Tit} - C_{Tit}) + B^*_{it-1} - \frac{B^*_{it}}{R^*_t}\right) + \mu_t B^*_{it} \Bigg\} \mathrm{d}i,$$

and take the second-order approximation around the global first-best allocation

$$\begin{aligned}\mathcal{L} - \hat{\mathcal{L}} &= -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \Bigg\{ \gamma \left(\frac{C_{Tit} - \hat{C}_{Tit}}{\bar{C}_T}\right)^2 + (1-\gamma)\left(\frac{C_{Nit} - \hat{C}_{Nit}}{\bar{C}_N}\right)^2 \\ &\qquad + 2\frac{P^*_{Tt} - \hat{P}^*_{Tt}}{\bar{P}^*_T}\frac{C_{Tit} - \hat{C}_{Tit}}{\bar{C}_T} - 2\frac{(B^*_{it} - \hat{B}^*_{it})(R^*_t - \hat{R}^*_t)}{\bar{R}^{*2}\bar{P}^*_T\bar{C}_T} \Bigg\} \mathrm{d}i + h.o.t. \\ &= -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \Big\{ \gamma \hat{z}^2_{it} + (1-\gamma) x^2_{it} \Big\} \mathrm{d}i + h.o.t.,\end{aligned}$$

where we used the fact that $\int_0^1 (C_{Tit} - \hat{C}_{Tit})\mathrm{d}i = \int_0^1 (B^*_{it} - \hat{B}^*_{it})\mathrm{d}i = 0.$ ■

Proof of Proposition 7 **Part (a)** can be verified directly, as $\psi_{it} = \bar{\omega}_i \bar{\sigma}^2_{it}(n^*_{it} + f^*_{it} - b^*_{it}) = 0$ for every $i \in (0, 1]$ ensures $\bar{\psi}_t = 0$, and hence $r^*_t = \hat{r}^*_t$ and a globally optimal tradable consumption allocation with $\mathbb{E}_t \Delta c_{it+1} = \mathbb{E}_t \Delta \hat{c}_{it+1} = \hat{r}^*_t$ for all i and t.

Further, unconstrained monetary policy in the US and other countries ensures efficient allocation in non-tradable sectors of all countries. Specifically, nominal dollar rate $i^*_t = \mathbb{E}_t \Delta a_{0t+1}$ in the US ensures $x_{0t} = c_{N0t} - a_{0t} = 0$, while nominal rates in other countries $i^*_{it} = \log(R_{it}/\bar{R}) = \mathbb{E}_t \Delta a_{it+1}$ ensure $x_{it} = c_{Nit} - a_{it} = 0$ in $i \in (0, 1]$, with the ensuing nominal exchange rate, $e_{it} = a_{it} - \hat{c}_{it} - p^*_{Tt}$. Finally, global market clearing implement $p^*_{Tt} = a_{0t} - \hat{c}_{0t}$ with associated expected inflation $\mathbb{E}_t \pi^*_{Tt+1} = i^*_t - \mathbb{E}_t \Delta y_{Tt+1}$, given that $\mathbb{E}_t \Delta c_{it+1} = \mathbb{E}_t \Delta \hat{c}_{it+1} = \mathbb{E}_t \Delta y_{Tt+1}$ for all i. As a result, the realized real interest rate $r^*_t = i^*_t - \mathbb{E}_t \pi^*_{Tt+1} = \mathbb{E}_t \Delta y_{Tt+1} = \hat{r}^*_t$, and our conjecture solution with $\hat{z}_{it} = c_{it} - \hat{c}_{it} = 0$ is verified.[58]

Part (b): Taking the path of monetary policy and output gaps $\{x_{it}\}$ as given, we focus on the optimal FX policy and the resulting allocation in the tradable sector. Therefore, given the characterization

[58] If a subset of countries $i \in (0, m_0]$ have a constrained monetary policy by fixed exchange rate, $e_{it} = 0$, then one can verify that the constrained global optimal allocation is still $\hat{z}_{it} = 0$ for all $i \in [0, 1]$, $x_{it} = 0$ for $i \in (m_0, 1]$, and $x_{it} = \hat{c}_{it} + p^*_{Tt} - a_{it}$ for $i \in [0, m_0]$, where the latter condition ensures $e_{it} = c_{Nit} - \hat{c}_{it} - p^*_{Tt} = 0$. If the US cooperates with the global planner, then US monetary policy chooses the path of i^*_t, and hence of p^*_{Tt} that implements the same real rate $r^*_t = \hat{r}^*_t$ and minimizes $\int_0^{m_0} x^2_{it}\mathrm{d}i$ period by period. If US does not cooperate, then it chooses i_t and p^*_{Tt} as above to close its own output gap, $x_{0t} = 0$, leaving the pegged countries with $x_{it} = \hat{c}_{it} + p^*_{Tt} - a_{it} = (a_{0t} - \hat{c}_{0t}) - (a_{it} - \hat{c}_{it})$ for $i \in (0, m_0]$, and the global planner cannot improve upon this allocation.

in Lemma 1, we can focus on the following subproblem:

$$\min_{\{\hat{z}_{it},b^*_{it},\psi_{it},\bar{\psi}_t\}} \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \hat{z}_{it}^2 \mathrm{d}i$$

$$\text{subject to} \qquad \beta b^*_{it} - b^*_{it-1} = -\hat{z}_{it} \qquad (\lambda_{it}),$$

$$\mathbb{E}_t \Delta \hat{z}_{it+1} = \psi_{it} - \bar{\psi}_t \qquad (\mu_{it}),$$

$$\int_0^1 \hat{z}_{it} \mathrm{d}i = 0 \qquad (\eta_t),$$

$$\bar{\psi}_t = \int_0^1 \psi_{it} \mathrm{d}i \qquad (\nu_t),$$

optimizing with respect to ψ_{it} for countries with unconstrained FXI f^*_{it} and taking ψ_{it} as exogenously given for countries with constrained FXI or fixed exchange rate ($\psi_{it} = 0$ for the latter as $\bar{\sigma}^2_{it} = 0$).

The cooperative policymaker has the following optimality conditions:

$$\begin{aligned}
\hat{z}_{it} &= \mu_{it} - \beta^{-1}\mu_{it-1} + \lambda_{it} + \eta_t, \\
\lambda_{it} &= \mathbb{E}_t \lambda_{it+1}, \\
\nu_t &= \bar{\mu}_t \equiv \int_0^1 \mu_{it} \mathrm{d}i, \\
\mu_{it} &= \nu_t = \bar{\mu}_t,
\end{aligned}$$

where the last FOC holds only for the subset of unconstrained countries $i \in (m_0 + m_1, 1]$, while countries $i \in [0, m_0]$ are pegged to the dollar (and, hence, have $\psi_{it} = 0$), and countries $i \in (m_0, m_0 + m_1]$ are non-pegged and constrained with some exogenous ψ_{it}. The unconstrained countries choose ψ_{it} to ensure $\mu_{it} = \bar{\mu}_t$, while the constrained countries face an exogenous ψ_{it} and a corresponding μ_{it}.

Writing the first FOC in expected differences and using the second FOC to eliminate $\mathbb{E}_t \Delta \lambda_{it+1} = 0$:

$$\mathbb{E}_t \Delta \mu_{it+1} - \beta^{-1} \Delta \mu_{it} + \mathbb{E}_t \Delta \eta_{t+1} = \mathbb{E}_t \Delta \hat{z}_{it+1} = \psi_{it} - \bar{\psi}_t,$$

where the last equality is the risk-sharing constraint. Integrating over i, we have:

$$(1+\beta)\bar{\mu}_t - \beta \mathbb{E}_t \bar{\mu}_{t+1} - \bar{\mu}_{t-1} = \beta \mathbb{E}_t \Delta \eta_{t+1}.$$

The initial condition is $\bar{\mu}_{-1} = \mu_{i,-1} = 0$ for all i by construction. Conjecture $\mathbb{E}_t \Delta \eta_{t+1} = 0$, so that $\mathbb{E}_0 \eta_t = \eta_0$ for all t, as risk-sharing conditions imply expected market clearing at all $t > 0$ provided market clearing at $t = 0$. Under this conjecture, the solution is $\bar{\mu}_t = 0$ for all $t \geq 0$.

By consequence, $\mu_{it} = \bar{\mu}_t = 0$ for all unconstrained countries i, and hence

$$\psi_{it} = \bar{\psi}_t = m_0 \cdot 0 + m_1 \bar{\psi}^c_t + (1 - m_1 - m_0)\bar{\psi}_t = \frac{m_1}{m_0 + m_1} \bar{\psi}^c_t,$$

where $\bar{\psi}_t^c \equiv \frac{1}{m_1}\int_{m_0}^{m_0+m_1}\psi_{it}\mathrm{d}i$ is the average UIP deviation for constrained countries. We have:

$$\mathbb{E}_t\Delta\hat{z}_{it+1} = \begin{cases} -\frac{m_1}{m_0+m_1}\bar{\psi}_t^c, & i\in[0,m_0],\\ +\frac{m_0}{m_0+m_1}\bar{\psi}_t^c, & \text{on average for } i\in(m_0,m_0+m_1],\\ 0, & i\in(m_0+m_1,1],\end{cases}$$

and, therefore, the conjectured solution implies $\mathbb{E}_0\int_0^1\hat{z}_{it}\mathrm{d}i = 0$ for all $t\geq 0$, which is necessary for market clearing, thus verifying our conjecture that $\mathbb{E}_0\eta_t = \eta_0$.

Note how this solution differs from the non-cooperative solution in which $\psi_{it} = 0$ for unconstrained countries, which results in $\bar{\psi}_t^n = m_1\bar{\psi}_t^c = \int_{m_0}^{m_0+m_1}\psi_{it}\mathrm{d}i$. For the same value of $\bar{\psi}_{it}^c > 0$, the non-cooperative solution results in a small interest rate wedge relative to the cooperative wedge, $\bar{\psi}_t^n = \hat{r}_t^* - r_t^* < \bar{\psi}_t$, but at the same time larger capital outflows from the constrained economies, which go equally to all countries $i\in[0,m_0]\cup(m_0+m_1,1]$ that now all have $\psi_{it}=0$ and $\mathbb{E}_t\Delta\hat{z}_{it+1} = -\bar{\psi}_t^n$. In contrast, in cooperative solution, unconstrained countries $i\in(m_0+m_1,1]$ lean against these capital inflows, resulting in $\mathbb{E}_t\Delta\hat{z}_{it+1}=0$. This curbs total capital outflows from constrained economies $i\in(m_0,m_0+m_1]$, but at the cost of a larger interest rate wedge $\bar{\psi}_t > \bar{\psi}_t^n > 0$ and large capital inflows to the US and pegged countries, $\mathbb{E}_t\Delta\hat{z}_{it+1} = -\bar{\psi}_t < -\bar{\psi}_t^n < 0$ for $i\in[0,m_0]$. ■

A5 Derivations and Proofs for Sections 5

A5.1 Staggered prices (Section 5.1)

The derivation of the NKPC and the loss function in the presence of inflation follows the standard steps. Using the property of the model that monetary policy affects exchange rates only via $\bar{\sigma}_t^2$, the planner's problem can be partitioned in two steps. The first one solves for the optimal trade-off between output gap and inflation. Because of the certainty equivalence and only first-period innovations affecting $\bar{\sigma}_t^2$, it is sufficient to focus on the following problem:

$$\begin{aligned} \min_{\{x_t,\pi_{Nt}\}} \quad & \frac{1}{2}\sum_{t=0}^{\infty}\beta^t(x_t^2+\alpha\pi_{Nt}^2)\\ \text{s.t.} \quad & \pi_{Nt} = \kappa x_t + \beta\pi_{Nt+1} + \nu_t,\\ & x_0 + \pi_{N0} = m_t. \end{aligned}$$

Taking the first-order conditions, we get

$$\beta^t x_t = \kappa\lambda_t + \mu_t,$$

$$\beta^t\alpha\pi_{Nt} = -\lambda_t + \lambda_{t-1}\beta + \mu_t,$$

where $\mu_t = 0$ for $t>0$ and $\lambda_{-1}=0$. It follows that the optimality conditions are

$$\alpha\kappa\pi_{Nt} = -x_t + x_{t-1}$$

for $t \geq 1$ and

$$\alpha\kappa\pi_{Nt} = -x_t + (1+\kappa)\mu_t,$$

for $t = 0$. Substitute the optimality condition into the NKPC, so that dynamics for $t > 0$ is given by

$$\beta x_{t+1} - \big(1 + \beta + \alpha\kappa^2\big)x_t + x_{t-1} = \alpha\kappa\nu_t.$$

This difference equation has two roots $\lambda_1 > 1$ and $\lambda_2 < 1$

$$\lambda_{1,2} = \frac{1}{2\beta}\left[1 + \beta + \alpha\kappa^2 \pm \sqrt{(1+\beta+\alpha\kappa^2)^2 - 4\beta}\right],$$

and assuming for simplicity that ν_t follows an AR(1) process, we get

$$x_t = \lambda_2 x_{t-1} - \frac{\alpha\kappa}{\beta}\frac{1}{\lambda_1 - \rho}\nu_t.$$

This means that one initial condition x_0 is required. At the same time, the NKPC for the first period together with the initial condition imply that

$$\alpha\kappa(m_t - x_0) = \alpha\kappa^2 x_0 - \beta\Delta x_1 + \alpha\kappa\varepsilon_{\nu 0}.$$

Substitute in expression for x_1 and solve for

$$x_0 = \frac{\alpha\kappa}{\alpha\kappa^2 + \alpha\kappa + \beta - \beta\lambda_2}\left[m_t - \frac{\lambda_1}{\lambda_1 - \rho}\varepsilon_{\nu 0}\right].$$

Substituting this result into equation for x_t, we get

$$x_t = k^x_{xt} m_t - k^x_{\nu t}\varepsilon_{\nu 0},$$

$$\pi_{Nt} = k^\pi_{xt} m_t - k^\pi_{\nu t}\varepsilon_{\nu 0}$$

for some coefficients k. Substitute this back into the objective function:

$$\sum_{t=0}^{\infty}\beta^t(x_t^2 + \alpha\pi_{Nt}^2) = \sum_{t=0}^{\infty}\beta^t\Big[(k^x_{xt}m_t - k^x_{\nu t}\varepsilon_{\nu 0})^2 + \alpha(k^\pi_{xt}m_t - k^\pi_{\nu t}\varepsilon_{\nu 0})^2\Big]$$

$$= \mathcal{K}_x m_t^2 + \mathcal{K}_\nu \varepsilon_{\nu 0}^2 + \mathcal{K}_{x\nu} m_t \varepsilon_{\nu 0} = k_1(m_t - k_2\varepsilon_{\nu 0})^2 + k_3\varepsilon_{\nu 0}^2.$$

Substitute solution from the first step keeping in mind that it holds for every innovation $\varepsilon_{\nu 0}$ to get the second-stage problem, which is largely isomorphic to the baseline model:

$$\begin{aligned}
\min_{\{z_t, m_t, b_t^*, f_t^*, \bar{\sigma}_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[\gamma z_t^2 + (1-\gamma)k_1(m_t - k_2\varepsilon_{\nu t})^2\Big] \\
\text{s.t.} \quad & \mathbb{E}_t\Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2\big(n_t^* + f_t^* - b_t^*\big), \\
& \beta b_t^* = b_{t-1}^* - z_t, \\
& \bar{\sigma}_t^2 = \mathrm{var}_t\left(\tilde{q}_{t+1} - z_{t+1} + m_{t+1}\right).
\end{aligned}$$

Going back to the policy in the non-tradable sector, consider whether the price level converges to the initial level in the long run. The optimal policy implements $\alpha\kappa\pi_{Nt} = -\Delta x_t$ for $t \geq 1$, just as in a closed economy. However, in the latter case, this condition holds also for $t = 0$ (under timeless perspective), which implies that $\alpha\kappa p_{Nt} = -x_t$ in all periods and given that x_t is stationary, the price level converges in the long run to the initial level. In contrast, in our model $\alpha\kappa p_{Nt} = -x_t + (x_0 + \alpha\kappa\pi_0)$ and given that $x_t \to 0$ in the long run, we get $p_{Nt} \to \frac{1}{\alpha\kappa}x_0 + \pi_0$, which is generically not equal zero.

A5.2 Terms of trade and incomplete pass-through (Section 5.2)

Loss function To derive the loss function, follow the same steps as in the baseline model. Write down the Lagrangian of the relaxed problem without nominal or financial frictions:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ (1-\gamma)\log C_{Ht} + \gamma \log C_{Ft} - L_t \\ + \lambda_t \Big(A_t L_t - C_{Ht} - \gamma P_{Ht}^{*-\varepsilon} C_t^* \Big) + \mu_t \left[B_{t-1}^* + \gamma P_{Ht}^{*1-\varepsilon} C_t^* - C_{Ft} - \frac{B_t^*}{R_t^*} \right] \Bigg\}.$$

Notice that the planner is allowed to set optimal price in foreign market and, in equilibrium, charges a constant markup $\frac{\varepsilon}{\varepsilon-1}$ over domestic price for the same goods. Take the first-order conditions and solve for the steady-state values of the Lagrange multipliers: $\bar{\lambda} = 1/A$, $\bar{\mu} = \left(\frac{\varepsilon}{\varepsilon-1}\frac{\bar{C}^*}{A}\right)^{\frac{\varepsilon-1}{\varepsilon}} \frac{1}{\bar{C}^*}$. Using these values and Lemma A2, derive quadratic loss function:

$$\mathcal{L} - \tilde{\mathcal{L}} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ (1-\gamma)(c_{Ht} - \tilde{c}_{Ht})^2 + \gamma(c_{Ft} - \tilde{c}_{Ft})^2 + \gamma(\varepsilon - 1)(p_{Ht}^* - \tilde{p}_{Ht}^*)^2 \Big\} + h.o.t., \quad \text{(A10)}$$

where as before, the small letters denote the deviations from the first-best allocation.

DCP The dollar pricing implies that P_{Ht}^* is fixed and therefore,

$$p_{Ht}^* = -\tilde{p}_{Ht}^* = \tilde{c}_{Ht} - \tilde{c}_{Ft} = \tilde{q}_t,$$

where $\tilde{q}_t$ is the natural real exchange rate. As before, define output gap as deviations from the optimal production of locally consumed goods $x_t \equiv c_{Ht} - \tilde{c}_{Ht}$ and the risk-sharing wedge as the deviation from the optimal consumption of foreign goods $z_t \equiv c_{Ft} - \tilde{c}_{Ft}$ and write the loss function (A10) as $\frac{1}{2}\mathbb{E}\sum_{t=0}^{\infty}\beta^t \Big[(1-\gamma)x_t^2 + \gamma z_t^2 + \gamma(\varepsilon-1)\tilde{q}_t^2 \Big]$. The first-order approximation to the budget constraint is

$$\beta b_t^* = b_{t-1}^* - (\varepsilon - 1)\tilde{q}_t - z_t,$$

where $b_t^* \equiv \frac{B_t^* - \tilde{B}_t^*}{\bar{C}_F}$. Intuitively, when the natural real exchange rate depreciates, the export price become too high reducing exports relative to the efficient allocation. Normalizing N_t^* and F_t^* by $\bar{C}_F$

and defining $\bar{\omega} \equiv \omega \bar{C}_F/\beta$, the planner's problem can be written as

$$\begin{aligned} \min_{\{x_t, z_t, b_t^*, f_t^*, \bar{\sigma}_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[\gamma z_t^2 + (1-\gamma) x_t^2 + \gamma(\varepsilon-1)\tilde{q}_t^2\big] \\ \text{s.t.} \quad & \mathbb{E}_t \Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2 (n_t^* + f_t^* - b_t^*), \\ & \beta b_t^* = b_{t-1}^* - (\varepsilon-1)\tilde{q}_t - z_t, \\ & \bar{\sigma}_t^2 = \mathrm{var}_t\big(\tilde{q}_{t+1} + x_{t+1} - z_{t+1}\big). \end{aligned}$$

It follows that when $\tilde{q}_t = 0$, the first-best allocation with zero losses and $x_t = z_t = 0$ is implementable with monetary policy that pegs the nominal exchange rate $\bar{\sigma}_t^2 = 0$. When two policy instruments are available, the risk-sharing condition is not binding and the problem reduces to minimizing the losses subject to the budget constraint. Let $\beta^t \lambda_t$ denote the Lagrange multiplier and take the optimality conditions with respect to z_t and b_t^*: $\gamma z_t = \lambda_t,\ \lambda_t = \mathbb{E}_t \lambda_{t+1}$. Thus, the monetary policy closes the output gap $x_t = 0$ and the FX interventions close the UIP gap $\mathbb{E}_t \Delta z_{t+1} = 0$ by setting $f_t^* = b_t^* - n_t^*$. Notice that $b_t^* \neq 0$ in this case because $\tilde{q}_t$ creates deviations from the optimal net exports. The fact that exogenous shocks $(\varepsilon-1)\tilde{q}_t$ in the budget constraint do not change any optimality conditions also implies that Theorems 1 and 2 remain true and the second-best monetary policy partially stabilizes the exchange rate.

PCP When sticky in producer currency, the export price in the currency of destination is equal

$$P_{Ht}^* = \frac{\varepsilon}{\varepsilon-1}\frac{P_{Ht}}{\mathcal{E}_t} = \frac{\varepsilon}{\varepsilon-1}\frac{1-\gamma}{\gamma}\frac{C_{Ft}}{C_{Ht}},$$

where the latter equality follows from household demand for goods. Therefore,

$$p_{Ht}^* = c_{Ft} - c_{Ht}$$

and it is sufficient to close two gaps in the loss function (A10) to implement the efficient allocation. Linearizing the market clearing condition, we get

$$a_t + l_t = (1-\bar{\gamma})c_{Ht} + \bar{\gamma}(c_t^* - \varepsilon p_{Ht}^*),$$

where $\bar{\gamma} \equiv \frac{\gamma(\varepsilon-1)}{\varepsilon-\gamma}$ is the steady-state share of exports in total output. Notice that $\bar{\gamma} \to 0$ when $\varepsilon \to 1$ as the export tax converges to infinity in this limit. The last two equations can be solved to express c_{Ht} and p_{Ht}^* in terms of the normalized output gap $x_t \equiv \frac{l_t - \tilde{l}_t}{1+\bar{\gamma}(\varepsilon-1)}$ and the risk-sharing gap $z_t \equiv \frac{c_{Ft} - \tilde{c}_{Ft}}{1+\bar{\gamma}(\varepsilon-1)}$:

$$c_{Ht} - \tilde{c}_{Ht} = \varepsilon\bar{\gamma} z_t + x_t, \qquad p_{Ht}^* - \tilde{p}_{Ht}^* = (1-\bar{\gamma}) z_t - x_t.$$

Substitute these expressions into the loss function:

$$\begin{aligned}
\mathcal{L}-\tilde{\mathcal{L}} &= -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{(1-\gamma)\Big(\varepsilon\bar{\gamma}z_t+x_t\Big)^2+\gamma\Big(1+\bar{\gamma}(\varepsilon-1)\Big)^2 z_t^2+\gamma(\varepsilon-1)\Big((1-\bar{\gamma})z_t-x_t\Big)^2\right\}+h.o.t.\\
&= -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\Big[1-\gamma+\gamma(\varepsilon-1)\Big]x_t^2+\left[(1-\gamma)\varepsilon^2\bar{\gamma}^2+\gamma\Big(1+\bar{\gamma}(\varepsilon-1)\Big)^2+\gamma(\varepsilon-1)(1-\bar{\gamma})^2\right]z_t^2\right\}+h.o.t.\\
&= -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\Big(1+\gamma(\varepsilon-2)\Big)x_t^2+\gamma\varepsilon^2 z_t^2\right\}+h.o.t.
\end{aligned}$$

It follows that the objective function can be expressed as $\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[(1-\gamma)x_t^2+\gamma\kappa z_t^2\Big]$, where $\kappa\equiv\frac{(1-\gamma)\varepsilon^2}{1+\gamma(\varepsilon-2)}>1$. Intuitively, when export prices are sticky in the currency of exporter, the monetary policy can generate expenditure switching in the market of destination and simultaneously close the output gap in domestic and export sectors. As a result, the loss function can be written in terms of the (total) output gap x_t and the deviations of imports from the optimal level z_t.

Linearizing the budget constraint and substituting in expression for p^*_{Ht}, we get

$$\beta b_t^* = b_{t-1}^* + \frac{\varepsilon-1}{\varepsilon}x_t - z_t,$$

where $b_t^*\equiv\frac{B_t^*-\tilde{B}_t^*}{\varepsilon\bar{C}_F}$. Normalizing noise trader shocks N_t^* and FX interventions F_t^* by $\bar{C}_F$, we get the risk-sharing condition

$$\mathbb{E}_t\Delta z_{t+1} = \bar{\omega}\bar{\sigma}^2\Big(n_t^*+f_t^*-b_t^*\Big),$$

where $\bar{\omega}\equiv\frac{\omega\bar{C}_F}{\beta(1+\bar{\gamma}(\varepsilon-1))}$. As before, the nominal exchange rate is given by

$$e_t = c_{Ht}-c_{Ft} = \tilde{q}_t + x_t - (1-\bar{\gamma})z_t.$$

Combining these conditions, we get the planner's problem:

$$\begin{aligned}
\min_{\{x_t,z_t,b_t^*,f_t^*,\bar{\sigma}_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[(1-\gamma)x_t^2+\gamma\kappa z_t^2\right]\\
\text{s.t.}\quad & \mathbb{E}_t\Delta z_{t+1} = \bar{\omega}\bar{\sigma}_t^2(n_t^*+f_t^*-b_t^*),\\
& \beta b_t^* = b_{t-1}^* + \frac{\varepsilon-1}{\varepsilon}x_t - z_t,\\
& \bar{\sigma}_t^2 = \mathrm{var}_t\big(\tilde{q}_{t+1}+x_{t+1}-(1-\bar{\gamma})z_{t+1}\big).
\end{aligned}$$

The only substantial difference from the baseline problem (15) is that the monetary policy affects exports via expenditure switching channel and therefore, x_t appears in the country's budget constraint with a multiplier that depends on the elasticity of substitution ε. This additional channel does not change the main results about the first-best policies: When two instruments are available, the planner can implement efficient allocation by closing the output gap $x_t=0$ with interest rate policy and eliminating the risk-sharing wedge with the FX interventions $f_t^*=-n_t^*$. Moreover, the divine coincidence still holds when efficient real exchange rate is constant: by stabilizing the nominal exchange rate, monetary policy alone can close both wedges $x_t = z_t = 0$. A sufficient condition for $\tilde{q}_t = 0$ is that $r_t^* = 0$ and $a_t = c_t^*$ follow a random walk. Indeed, in this case $\tilde{c}_{Ft}$ is also a random walk and moves one-to-one with a_t, which given $\tilde{c}_{Ht} = a_t$ implies that $\tilde{q}_t = \tilde{c}_{Ht} - \tilde{c}_{Ft} = 0$.

Moving to the second-best policies, the effect of monetary policy on country's exports implies that a nominal peg $\sigma_t^2 = 0$ is no longer sufficient to implement $z_t = 0$. However, for any given path of x_t, it is still optimal to close the UIP deviations — either using the FX interventions or by stabilizing the nominal exchange rate. On top of that, there is a new channel through which monetary policy can affect z_t ex-post: the expenditure switching boosts exports that increase the supply of foreign currency b_t^* and can be used to satisfy noise trader demand.

Incomplete pass-through Generalize the baseline model in two ways. First, assume isoelastic separable preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\gamma C_{Tt}^{\frac{\theta-1}{\theta}} + (1-\gamma)(C_{Nt}^{\frac{\theta-1}{\theta}} - L_t)\Big],$$

where θ is the elasticity of substitution between tradable and non-tradable goods. Second, allow for pricing-to-market in the tradable sector: while foreign suppliers still charge $P_{Tt}^* = 1$ dollars, local retailers charge $P_{Tt} = (\mathcal{E}_t P_{Tt}^*)^{\alpha} P_{Nt}^{1-\alpha} = \mathcal{E}_t^{\alpha}$ units of local currency. It follows the goods market clearing condition (2) is replaced with

$$\frac{\gamma}{1-\gamma}\left(\frac{C_{Nt}}{C_{Tt}}\right)^{\frac{1}{\theta}} = \left(\frac{\mathcal{E}_t P_{Tt}^*}{P_{Nt}}\right)^{\alpha} = \mathcal{E}_t^{\alpha} = P_{Tt},$$

while the household Euler equation (3) remains unchanged. The profits and losses of the retail sector are redistributed lump-sum to households. All arbitrageurs and noise traders are local agents and, for tractability reasons, we assume that international bonds are denominated in units of tradable goods so that the carry-trade returns are given by $\tilde{R}_{t+1} = R_t^* - R_t \frac{P_{Tt}}{P_{Tt+1}}$. Combine the arbitrageurs' optimal portfolio choice (4) with the household Euler equation and the bonds market clearing condition (5) to obtain the international risk sharing

$$\beta R_t^* \mathbb{E}_t \left(\frac{C_{Tt}}{C_{Tt+1}}\right)^{\frac{1}{\theta}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \qquad \text{where} \quad \sigma_t^2 = R_t^2 \cdot \mathrm{var}_t \left(\frac{P_{Tt}}{P_{Tt+1}}\right).$$

The country's budget constraint (6) remains unchanged.

Because the feasibility constraints in the social planner's problem — the market clearing for non-tradables and the country's budget constraint — are the same as in the baseline model, the quadratic loss function does not depend on α and changes only because of more general preferences. Applying the result from Lemma A2, we get the second-order approximation to the objective function

$$\mathcal{L} - \hat{\mathcal{L}} = -\frac{1}{2}\frac{\theta-1}{\theta^2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \gamma \bar{C}_T^{\frac{\theta-1}{\theta}} \left(\frac{C_{Tt} - \tilde{C}_{Tt}}{\bar{C}_T}\right)^2 + (1-\gamma)\bar{C}_N^{\frac{\theta-1}{\theta}} \left(\frac{C_{Nt} - \tilde{C}_{Nt}}{C_N}\right)^2 \right\} + h.o.t.$$

Normalizing the steady-state values $\bar{C}_T = \bar{C}_N$ and taking the first-order approximation of the equilibrium conditions, we get the following policy problem:

$$
\begin{aligned}
\min_{\{x_t, f_t^*, z_t, p_{Tt}, b_t^*, \bar{\sigma}_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \beta^t \big[\gamma z_t^2 + (1-\gamma) x_t^2\big] \\
\text{subject to} \quad & \beta b_t^* = b_{t-1}^* - z_t, \\
& p_{Tt} = \tilde{q}_t + \frac{1}{\theta}(x_t - z_t), \\
& \mathbb{E}_t \Delta z_{t+1} = \theta \bar{\omega} \cdot \mathrm{var}_t(p_{Tt+1}) \cdot (n_t^* + f_t^* - b_t^*),
\end{aligned}
$$

where $\bar{\omega} \equiv \omega \bar{Y}_T / \beta$ and $\tilde{q}_t \equiv \frac{1}{\theta}(\tilde{c}_{Nt} - \tilde{c}_{Tt})$ is the first-best relative price of tradables and non-tradables. It follows that all results for the baseline model extend to this setup up to rescaling of $\tilde{q}_t$ and $\bar{\omega}$ by θ. In particular, the optimal allocation does not depend on the degree of pricing-to-market α. In contrast, the latter affects the equilibrium nominal exchange rate, which can be found from $e_t = \frac{1}{\alpha} p_{Tt}$. Thus, the incomplete pass-through $\alpha < 1$ and a low substitution between goods $\theta < 0$ can reconcile low observed volatility in output and consumption with high volatility of the exchange rate supporting this allocation.

References

Aiyagari, S. R., and M. Gertler (1999): "“Overreaction” of asset prices in general equilibrium," *Review of Economic Dynamics*, 2(1), 3–35.

Akinci, O., S. Kalemli-Özcan, and A. Queralto (2022): "Uncertainty Shocks, Capital Flows, and International Risk Spillovers," Discussion paper, National Bureau of Economic Research.

Alvarez, F., A. G. Atkeson, and P. J. Kehoe (2009): "Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium," *Review of Economic Studies*, 76(3), 851–878.

Amador, M., J. Bianchi, L. Bocola, and F. Perri (2019): "Exchange Rate Policies at the Zero Lower Bound," *The Review of Economic Studies*, 87(4), 1605–45.

Atkeson, A. G., V. V. Chari, and P. J. Kehoe (2010): "Sophisticated monetary policies," *The Quarterly journal of economics*, 125(1), 47–89.

Atkeson, A. G., A. L. Eisfeldt, and P.-O. Weill (2015): "Entry and Exit in OTC Derivatives Markets," *Econometrica*, 83(6), 2231–92.

Bacchetta, P., K. Benhima, and B. Berthold (2023): "Foreign exchange intervention with UIP and CIP deviations: The case of small safe haven economies," *Swiss Finance Institute Research Paper*, (23-71).

Bacchetta, P., S. Tieche, and E. Van Wincoop (2020): "International portfolio choice with frictions: Evidence from mutual funds," *Swiss Finance Institute Research Paper*, (20-46).

Bahaj, S., and R. Reis (2021): "Central Bank Swap Lines: Evidence on the Effects of the Lender of Last Resort," *The Review of Economic Studies*, 89(4), 1654–93.

——— (2023): "The anatomy of a peg: lessons from China's parallel currencies," .

Basu, S. S., E. Boz, G. Gopinath, F. Roch, and F. D. Unsal (2020): "A Conceptual Model for the Integrated Policy Framework," IMF Working Papers No. 2020/121.

Bekaert, G. (1995): "The time variation of expected returns and volatility in foreign-exchange markets," *Journal of Business & Economic Statistics*, 13(4), 397–408.

Beltran, P., and C. He (2023): "Inelastic Financial Markets and Foreign Exchange Interventions," working paper.

Benigno, G., and P. Benigno (2003): "Price stability in open economies," *The Review of Economic Studies*, 70(4), 743–764.

Benigno, P. (2009): "Price stability with imperfect financial integration," *Journal of Money, credit and Banking*, 41, 121–149.

Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2017): "Fiscal policy and debt management with incomplete markets," *The Quarterly Journal of Economics*, 132(2), 617–663.

Bianchi, J. (2011): "Overborrowing and systemic externalities in the business cycle," *American Economic Review*, 101(7), 3400–3426.

Bianchi, J., and L. Coulibaly (2023): "A Theory of Fear of Floating," Discussion paper, National Bureau of Economic Research.

Blanchard, O., and F. Giavazzi (2002): "Current Account Deficits in the Euro Area: The End of the Feldstein Horioka Puzzle?," *Brookings Papers on Economic Activity*, 33(2), 147–210.

Blanchard, O. J., F. Giavazzi, and F. Sa (2005): "International Investors, the U.S. Current Account, and the Dollar," *Brookings Papers on Economic Activity*, 36(1), 1–66.

Caballero, R., and E. Farhi (2018): "The Safety Trap," *The Review of Economic Studies*, 85(1), 223–74.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008): "An Equilibrium Model of “Global Imbalances” and Low Interest Rates," *American Economic Review*, 98(1), 358–393.

Caballero, R. J., and A. Simsek (2022): "A Monetary Policy Asset Pricing Model," Discussion paper, NBER Working Paper No. 30132.

Calvo, G. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.

Calvo, G. A., and C. M. Reinhart (2002): "Fear of Floating*," *The Quarterly Journal of Economics*, 117(2), 379–408.

Camanho, N., H. Hau, and H. Rey (2022): "Global portfolio rebalancing and exchange rates," *The Review of Financial Studies*, 35(11), 5228–5274.

Caramp, N., and D. H. Silva (2023): "Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity," working paper.

Cavallino, P. (2019): "Capital Flows and Foreign Exchange Intervention," *American Economic Journal: Macroeconomics*, 11(2), 127–70.

Clarida, R., J. Gali, and M. Gertler (1999): "The science of monetary policy: a new Keynesian perspective," *Journal of economic literature*, 37(4), 1661–1707.

Cole, H. L., and M. Obstfeld (1991): "Commodity trade and international risk sharing," *Journal of Monetary Economics*, 28(1), 3–24.

Corsetti, G., L. Dedola, and S. Leduc (2010): "Optimal Monetary Policy in Open Economies," in *Handbook of Monetary Economics*, ed. by B. M. Friedman, and M. Woodford, vol. 3, chap. 16, pp. 861–933. Elsevier.

Corsetti, G., and P. Pesenti (2001): "Welfare and Macroeconomic Interdependence," *The Quarterly Journal of Economics*, 116(2), 421–45.

Costinot, A., G. Lorenzoni, and I. Werning (2014): "A theory of capital controls as dynamic terms-of-trade manipulation," *Journal of Political Economy*, 122(1), 77–128.

Das, M., G. Gopinath, and S. Kalemli-Özcan (2022): "Preemptive policies and risk-off shocks in emerging markets," Discussion paper, National Bureau of Economic Research.

Dávila, J., J. Hong, P. Krusell, and J.-V. Ríos-Rull (2012): "Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks," *Econometrica*, 80(6), 2431–2467.

Devereux, M. B., and C. Engel (2002): "Exchange rate pass-through, exchange rate volatility, and exchange rate disconnect," *Journal of Monetary Economics*, 49(5), 913–940.

——— (2003): "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility," *Review of Economic Studies*, 70, 765–84.

Dornbusch, R. (1976): "Expectations and Exchange Rate Dynamics," *Journal of Political Economy*, 84(6), 1161–76.

——— (1988): "Real exchange rates and macroeconomics: a selective survey," NBER, Cambridge MA, USA.

Driskill, R., and S. McCafferty (1987): "Exchange-rate determination: An equilibrium approach with imperfect capital substitutability," *Journal of International Economics*, 23(3-4), 241–261.

Egorov, K., and D. Mukhin (2023): "Optimal policy under dollar pricing," *American Economic Review*, 113(7), 1783–1824.

Eichengreen, B. J., and M. Flandreau (1997): *The gold standard in theory and history*. Psychology press.

Engel, C. (1996): "The forward discount anomaly and the risk premium: A survey of recent evidence," *Journal of Empirical Finance*, 3(2), 123–192.

——— (2011): "Currency misalignments and optimal monetary policy: a reexamination," *The American Economic Review*, 101(6), 2796–2822.

——— (2016): "Exchange Rates, Interest Rates, and the Risk Premium," *American Economic Review*, 106(2), 436–474.

Fanelli, S. (2017): "Optimal Monetary Policy, Capital Controls, and International Portfolios," working paper.

Fanelli, S., and L. Straub (2021): "A Theory of Foreign Exchange Interventions," *The Review of Economic Studies*, 88(6), 2857–85.

Farhi, E., G. Gopinath, and O. Itskhoki (2014): "Fiscal Devaluations," *Review of Economics Studies*, 81(2), 725–760.

Farhi, E., and I. Werning (2012): "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates," http://scholar.harvard.edu/farhi/publications.

——— (2016): "A theory of macroprudential policies in the presence of nominal rigidities," *Econometrica*, 84(5), 1645–1704.

——— (2017): "Fiscal unions," *American Economic Review*, 107(12), 3788–3834.

Fleming, J. M. (1962): "Domestic Financial Policies under Fixed and under Floating Exchange Rates," *IMF Staff Papers*, 9(3), 369–380.

Fornaro, L. (2022): "A Theory of Monetary Union and Financial Integration," *Review of Economic Studies*, 89(4), 1911–47.

Friedman, M. (1953): "The case for flexible exchange rates," *Essays in positive economics*, 157(203), 33.

——— (1969): *The optimum quantity of money: and other essays*. Aldine De Gruyter.

Fukui, M., E. Nakamura, and J. Steinsson (2023): "The Macroeconomic Consequences of Exchange Rate Depreciations," NBER Working Paper No. 31279.

Gabaix, X., and M. Maggiori (2015): "International Liquidity and Exchange Rate Dynamics," *The Quarterly*

Journal of Economics, 130(3), 1369–1420.
Gali, J., and T. Monacelli (2005): "Monetary policy and exchange rate volatility in a small open economy," *The Review of Economic Studies*, 72(3), 707–734.
Galí, J., and T. Monacelli (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72(3), 707–734.
Geanakoplos, J. D., and H. M. Polemarchakis (1986): "Existence, regularity, and constrained suboptimality of competitive allocations," *Essays in Honor of Kenneth J. Arrow: Volume 3, Uncertainty, Information, and Communication*, 3, 65.
Goldberg, L., and C. Tille (2009): "Macroeconomic interdependence and the international role of the dollar," *Journal of Monetary Economics*, 56(7), 990–1003.
Gopinath, G., E. Boz, C. Casas, F. J. Díez, P.-O. Gourinchas, and M. Plagborg-Møller (2020): "Dominant currency paradigm," *American Economic Review*, 110(3), 677–719.
Gopinath, G., and O. Itskhoki (2022): "Dominant Currency Paradigm: a review," in *Handbook of International Economics: International Macroeconomics, Volume 6*, ed. by G. Gopinath, E. Helpman, and K. Rogoff, vol. 6 of *Handbook of International Economics*, chap. 2, pp. 45–90. Elsevier.
Gourinchas, P.-O., W. Ray, and D. Vayanos (2019): "A preferred-habitat model of term premia and currency risk," Discussion paper, mimeo.
Greenwood, R., S. G. Hanson, J. C. Stein, and A. Sunderam (2020): "A Quantity-Driven Theory of Term Premia and Exchange Rates," Discussion paper, National Bureau of Economic Research.
Hansen, L. P., and J. Miao (2018): "Aversion to ambiguity and model misspecification in dynamic stochastic environments," *Proceedings of the National Academy of Sciences*, 115(37), 9163–9168.
Hansen, L. P., and T. J. Sargent (2011): "Robustness and ambiguity in continuous time," *Journal of Economic Theory*, 146(3), 1195–1223.
Hassan, T. A., T. M. Mertens, and T. Zhang (2023): "A risk-based theory of exchange rate stabilization," *The Review of Economic Studies*, 90(2), 879–911.
Hau, H., M. Massa, and J. Peress (2009): "Do Demand Curves for Currencies Slope Down? Evidence from the MSCI Global Index Change," *The Review of Financial Studies*, 23(4), 1681–717.
Ilzetzki, E., C. M. Reinhart, and K. S. Rogoff (2019): "Exchange Arrangements Entering the Twenty-First Century: Which Anchor will Hold?," *The Quarterly Journal of Economics*, 134(2), 599–646.
Itskhoki, O., and B. Moll (2019): "Optimal Development Policies with Financial Frictions," *Econometrica*, 87, 139–173.
Itskhoki, O., and D. Mukhin (2021a): "Exchange Rate Disconnect in General Equilibrium," *Journal of Political Economy*, 129, 2183–2232.
——— (2021b): "Mussa Puzzle Redux," NBER Working Paper No. 28950.
——— (2022): "Sanctions and the Exchange Rate," NBER Working Paper No. 30009.
Jeanne, O. (2012): "Capital Account Policies and the Real Exchange Rate," in *NBER International Seminar on Macroeconomics*, pp. 7–42. University of Chicago Press.
——— (2022): "Rounding the corners of the trilemma: A simple framework," *Journal of International Money and Finance*, 122, 102551.
Jeanne, O., and A. Korinek (2010): "Excessive volatility in capital flows: A pigouvian taxation approach," *American Economic Review*, 100(2), 403–07.
Jeanne, O., and A. K. Rose (2002): "Noise Trading and Exchange Rate Regimes," *The Quarterly Journal of Economics*, 117(2), 537–569.
Jiang, Z., A. Krishnamurthy, and H. N. Lustig (2021): "Foreign Safe Asset Demand and the Dollar Exchange Rate," *The Journal of Finance*, 76(3), 1049–1089.
Judd, K. L., and S.-M. Guu (2001): "Asymptotic methods for asset market equilibrium analysis," *Economic theory*, 18, 127–157.
Kalemli-Özcan, S., and L. Varela (2021): "Five Facts about the UIP Premium," NBER Working Paper No. 28923.
Kareken, J., and N. Wallace (1981): "On the indeterminacy of equilibrium exchange rates," *The Quarterly Journal of Economics*, 96(2), 207–222.
Kekre, R., and M. Lenel (2021): "The flight to safety and international risk sharing," Discussion paper, National Bureau of Economic Research.

——— (2022): "Monetary policy, redistribution, and risk premia," *Econometrica*, 90(5), 2249–2282.

Koijen, R. S. J., and M. Yogo (2020): "Exchange Rates and Asset Prices in a Global Demand System," NBER Working Papers No. 27342.

Kollmann, R. (2004): "Welfare Effects of a Monetary Union: The Role of Trade Openness," *Journal of the European Economic Association*, 2(2–3), 289–301.

——— (2005): "Macroeconomic effects of nominal exchange rate regimes: new insights into the role of price dynamics," *Journal of International Money and Finance*, 24(2), 275–292.

Kouri, P. (1983): "Balance of Payments and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model," in *Economic Interdependence and Flexible Exchange Rates*, ed. by J. S. Bhandari, and B. H. Putnam. MIT Press.

Krugman, P. R. (1979): "A Model of Balance-of-Payments Crises," *Journal of Money, Credit and Banking*, 11(3), 311–325.

——— (1991): "Target zones and exchange rate dynamics," *The Quarterly Journal of Economics*, 106(3), 669–682.

Marcet, A., and R. Marimon (2019): "Recursive contracts," *Econometrica*, 87(5), 1589–1631.

Marcet, A., and J. P. Nicolini (2003): "Recurrent Hyperinflations and Learning," *American Economic Review*, 93(5), 1476–98.

Mendoza, E. G., V. Quadrini, and J. Ríos-Rull (2009): "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy*, 117(3), 371–416.

Mundell, R. A. (1961): "A Theory of Optimum Currency Areas," *The American Economic Review*, 51(4), 657–65.

——— (1962): "The Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability," *IMF Staff Papers*, 9(1), 70–79.

——— (1963): "Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates," *The Canadian Journal of Economics and Political Science*, 29(4), 475–485.

Mussa, M. L. (1986): "Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications," *Carnegie-Rochester Conference Series on Public Policy*, 25(1), 117–214.

Obstfeld, M., and K. S. Rogoff (1995): "Exchange Rate Dynamics Redux," *Journal of Political Economy*, 103, 624–60.

——— (2001): "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," in *NBER Macroeconomics Annual 2000*, vol. 15, pp. 339–390.

——— (2002): "Global implications of self-oriented national monetary rules," *The Quarterly journal of economics*, 117(2), 503–535.

Patel, N., and P. Cavallino (2019): "FX intervention: goals, strategies and tactics," *BIS Paper*, (104b).

Ray, W. (2019): "Monetary policy and the limits to arbitrage: Insights from a New Keynesian preferred habitat model," in *2019 Meeting Papers*, vol. 692. Society for Economic Dynamics.

Rey, H. (2013): "Dilemma not trilemma: the global financial cycle and monetary policy independence," *Jackson Hole Symposium Proceedings*.

Rogoff, K. S. (1996): "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, 34, 647–668.

Schmitt-Grohé, S., and M. Uribe (2016): "Downward nominal wage rigidity, currency pegs, and involuntary unemployment," *Journal of Political Economy*, 124(5), 1466–1514.

Silva, D. H. (2016): "The Risk Channel of Unconventional Monetary Policy," Discussion paper.

Svensson, L. E. (1994): "Why exchange rate bands?: Monetary independence in spite of fixed exchange rates," *Journal of Monetary Economics*, 33(1), 157–199.

Wallace, N. (1981): "A Modigliani-Miller theorem for open-market operations," *The American Economic Review*, 71(3), 267–274.

Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.